Geometry Workbook Solutions

Circles

krista king MATH

EQUATION OF A CIRCLE

■ 1. A circle has a radius of 4 and center at $(-2,5)$. Write the equation for this circle.

Solution:

$(x+2)^2+(y=5)^2=16$. The general equation for a circle is $(x-h)^2+(y-k)^2=r^2$ where (h,k) is the center and r is the radius. $(-2,5)$ is the center, so $h=-2$ and $k=5$. The radius is given as $r=4$. Substitute these values into the general equation and get $(x+2)^2+(y-5)^2=16$.

■ 2. Find the center and diameter of the circle given by $(x-3)^2+(y+2)^2=9$.

Solution:

Center at $(3,-2)$, diameter of $d=6$. The general equation for a circle is $(x-h)^2+(y-k)^2=r^2$ where (h,k) is the center and r is the radius. In this equation, $h=3$ and $k=-2$ making the center $(3,-2)$. The radius is $\sqrt{9}=3$. Therefore, the diameter of the circle is 6.

© Krista King Math

■ 3. A circle has a diameter with endpoints at $(-3, -1)$ and $(3,7)$. Find the equation of the circle.

Solution:

$x^2 + (y-3)^2 = 25$. The diameter of the circle can be found by finding the distance between the points given. Use the distance formula to get

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(7-(-1))^2 + (3-(-3)^2}$$

$$d = \sqrt{100}$$

$$d = 10$$

Use the midpoint formula to find the center of the circle.

$$(a,b) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(a,b) = \left(\frac{(-3)+3}{2}, \frac{(-1)+7}{2}\right)$$

$$(a,b) = (0,3)$$

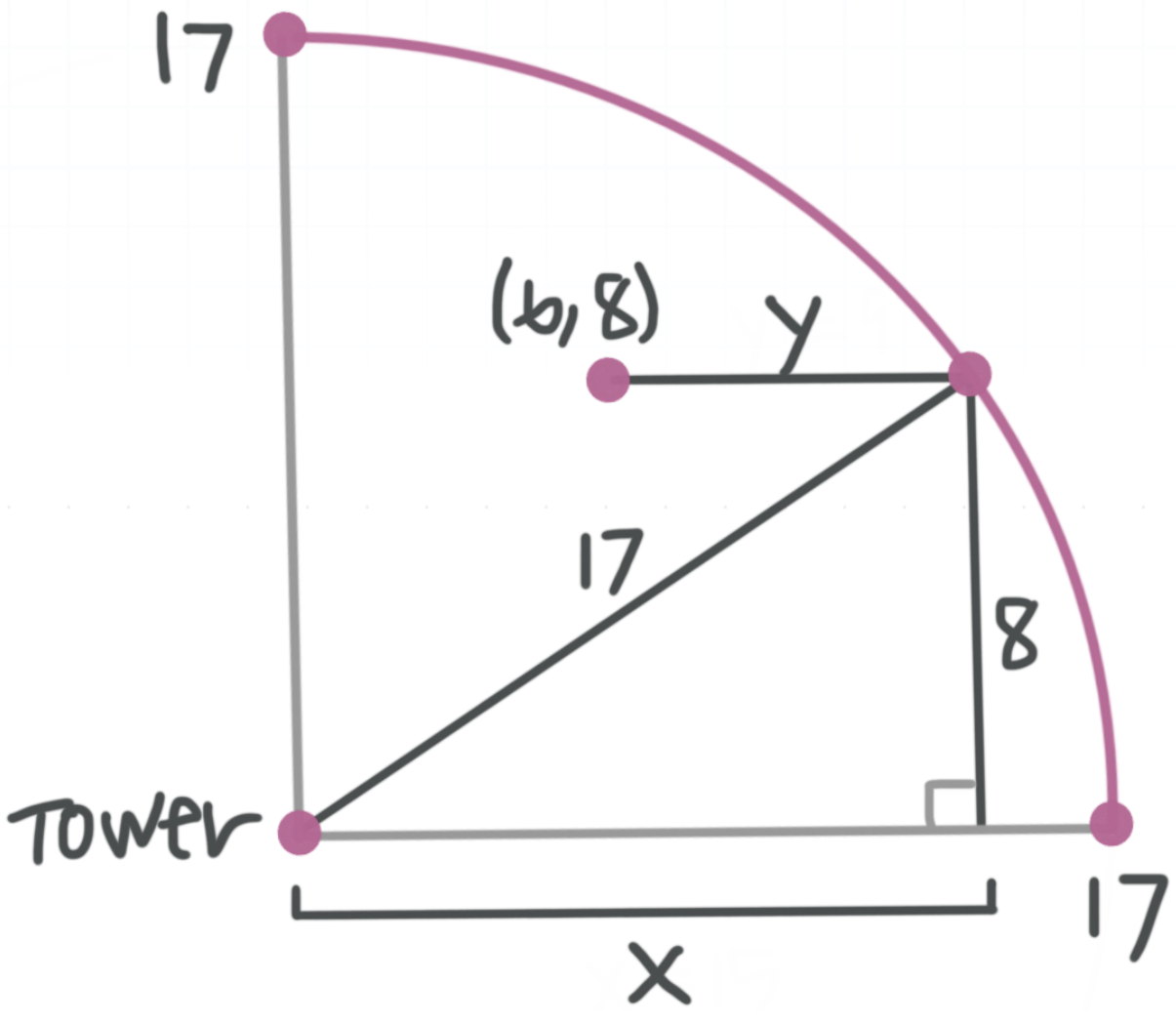
The general equation for a circle is $(x-h)^2 + (y-k)^2 = r^2$ where (h,k) is the center and r is the radius. Substitute $h = 0$, $k = 3$, and $r = 5$ into the equation to get $x^2 + (y-3)^2 = 25$.

© Krista King Math

■ 4. A cellphone tower services a 17 mile radius. A rest stop on the highway is 6 miles east and 8 miles north of the tower. If you continue to travel due east from the rest stop, for how many more miles will you be in range of the tower?

Solution:

9 miles. Sketch the following diagram.



Find the horizontal length of the right triangle shown and get $x^2 + 8^2 = 17^2$. Solve for x.

© Krista King Math

$$x^2 + 8^2 = 17^2$$

$$x^2 + 64 = 289$$

$$x^2 = 225$$

$$x = 15$$

Now find y.

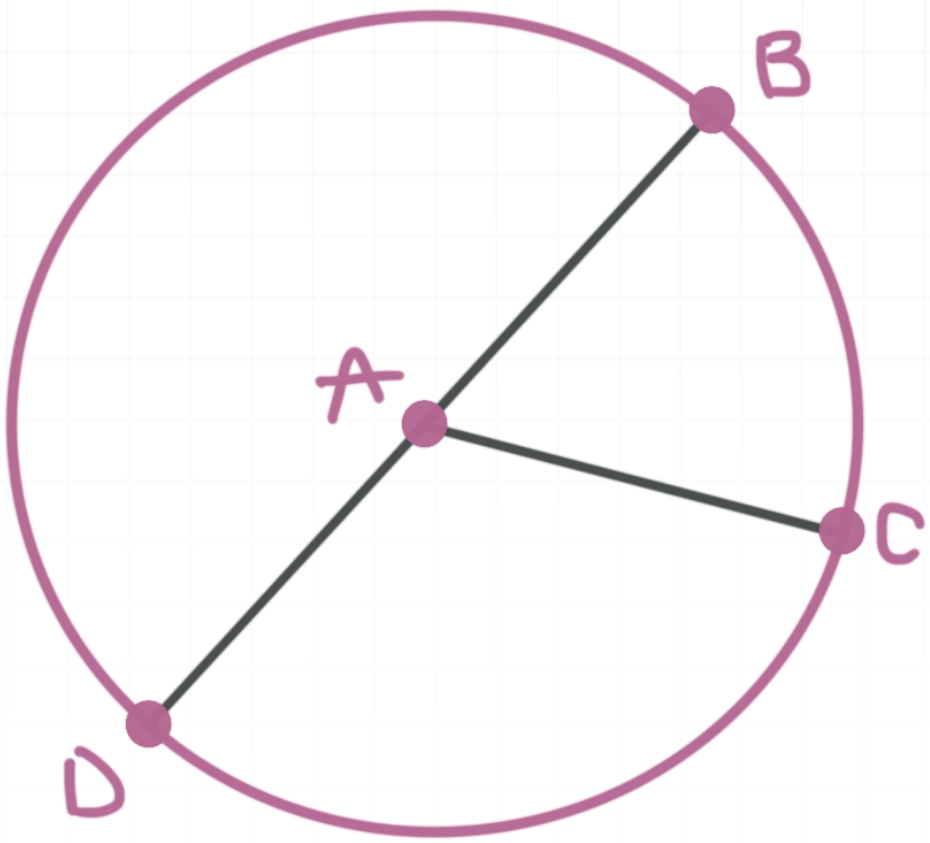
$$y = x - 6$$

$$y = 9$$

© Krista King Math

DEGREE MEASURE OF AN ARC

■ 1. In $\odot A$, $m\angle BAC = 65°$ and $\overline{BD}$ is a diameter. Find the measure of arc DC.



Solution:

$115°$. $\angle BAC$ is a central angle of $\odot A$, so arc BC is given by the measure of $\angle BAC$, and $m\angle BAC = 65°$. Since $\angle BAC$ and $\angle CAD$ are supplementary,

$$m\angle BAC + m\angle CAD = 180°$$

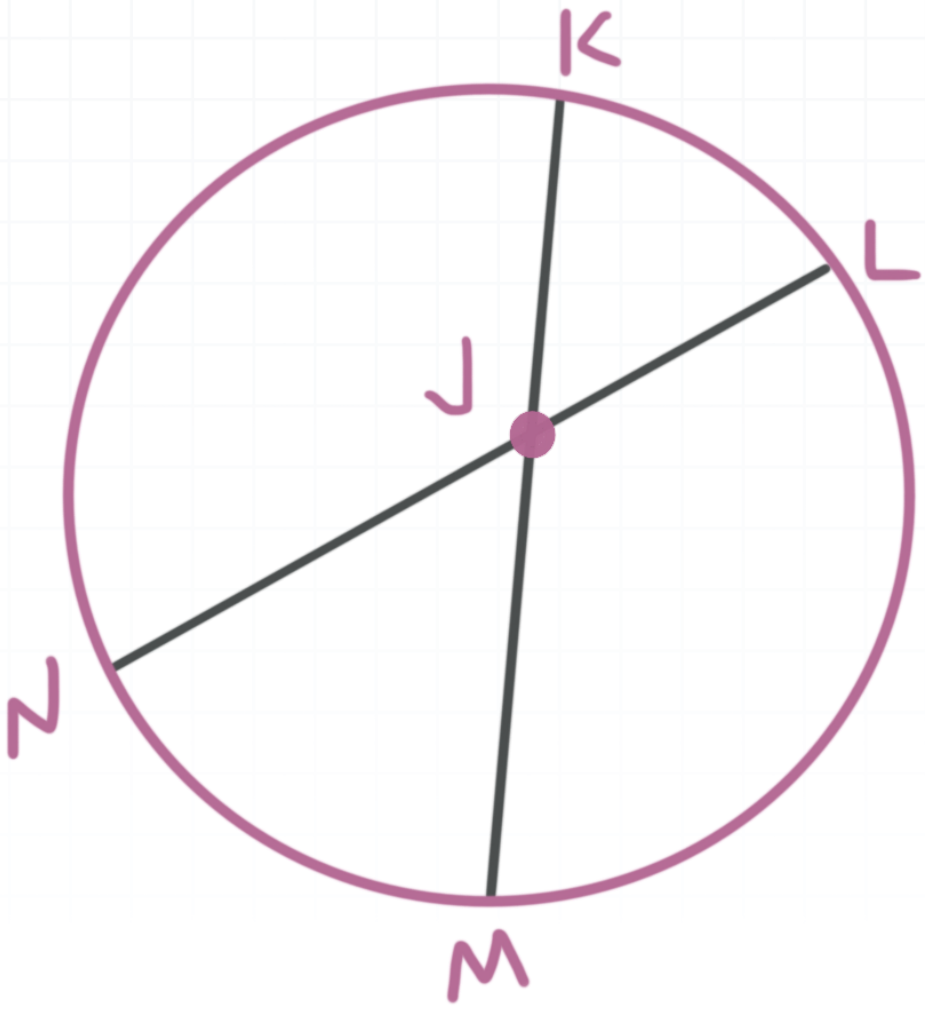
$$65° + m\angle CAD = 180°$$

$$m\angle CAD = 115°$$

Therefore, the measure of arc DC must be $115°$.

© Krista King Math

■ 2. In $\odot J$, $m\angle KJL = 54°$ and $\overline{KM}$ and $\overline{LN}$ are diameters. Find the measure of arc MN.

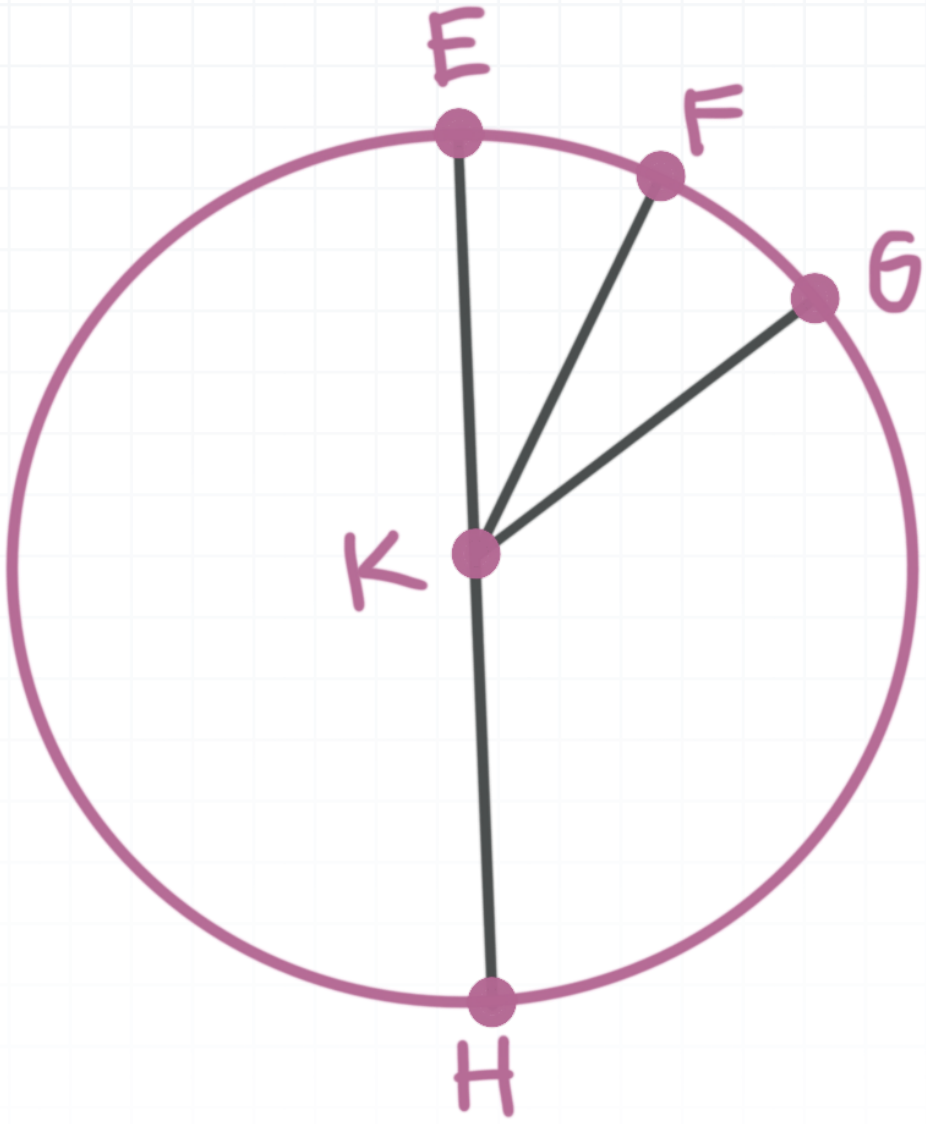


Solution:

$54°$. $\angle KJL$ is a central angle of $\odot J$. Because they are vertical angles, $m\angle KJL = m\angle NJM$. Because $m\angle KJL = 54°$, that means $m\angle NJM = 54°$, and therefore that the measure of arc MN is also $54°$.

■ 3. In $\odot K$, $m\angle EKG = 70°$, $\overline{EH}$ is a diameter, and $\overline{KF}$ bisects $\angle EKG$. Find the measure of arc FEH.

© Krista King Math

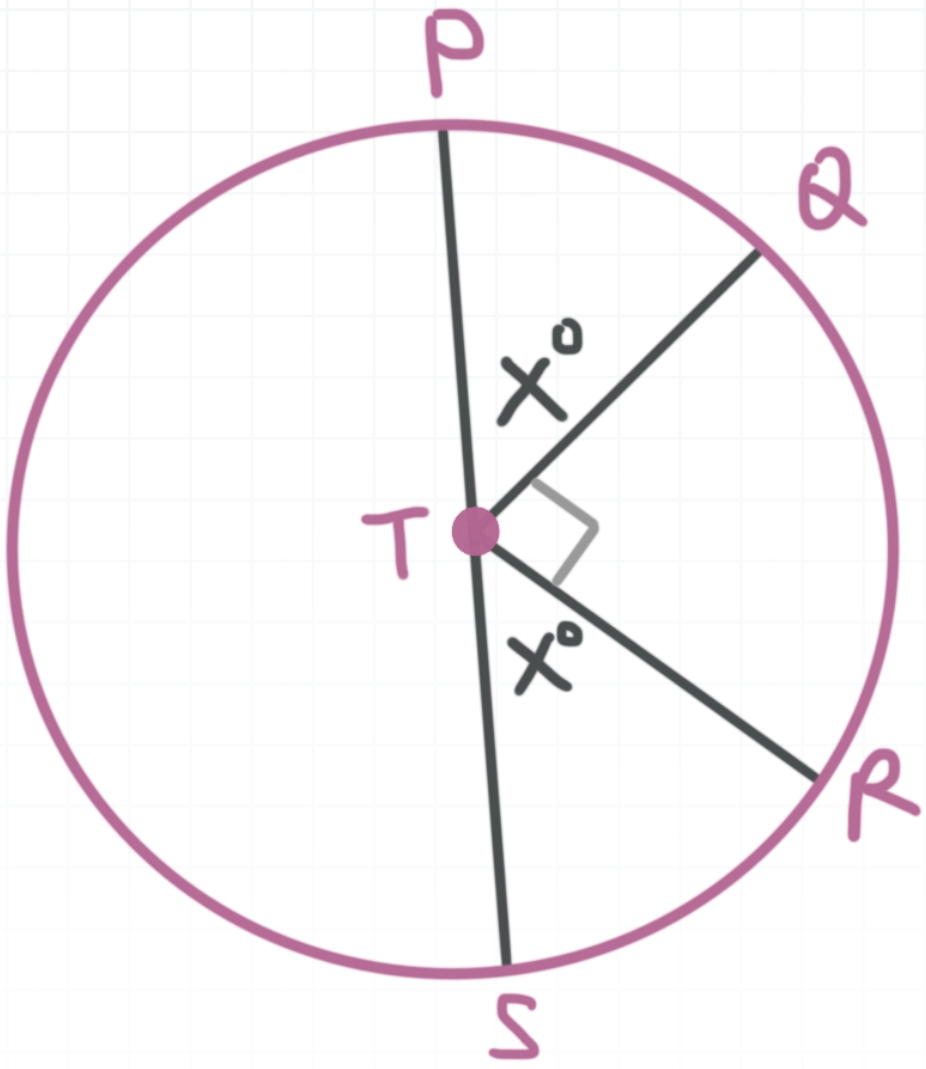


Solution:

$215°$. $\angle EKG = 70°$ and is bisected by KF, so $m\angle EKF$ and the measure of arc EF are both equal to $35°$. Arc FEH is the sum of arc EF and arc EH, so arc FEH must have a measure of $35° + 180° = 215°$.

■ 4. Find the measure of arc PR, if $\overline{PS}$ is the diameter of $\odot T$.

© Krista King Math



Solution:

$135°$. Because $\overline{PS}$ is a diameter,

$$m\angle PTQ + m\angle QTR + m\angle RTS = 180°$$

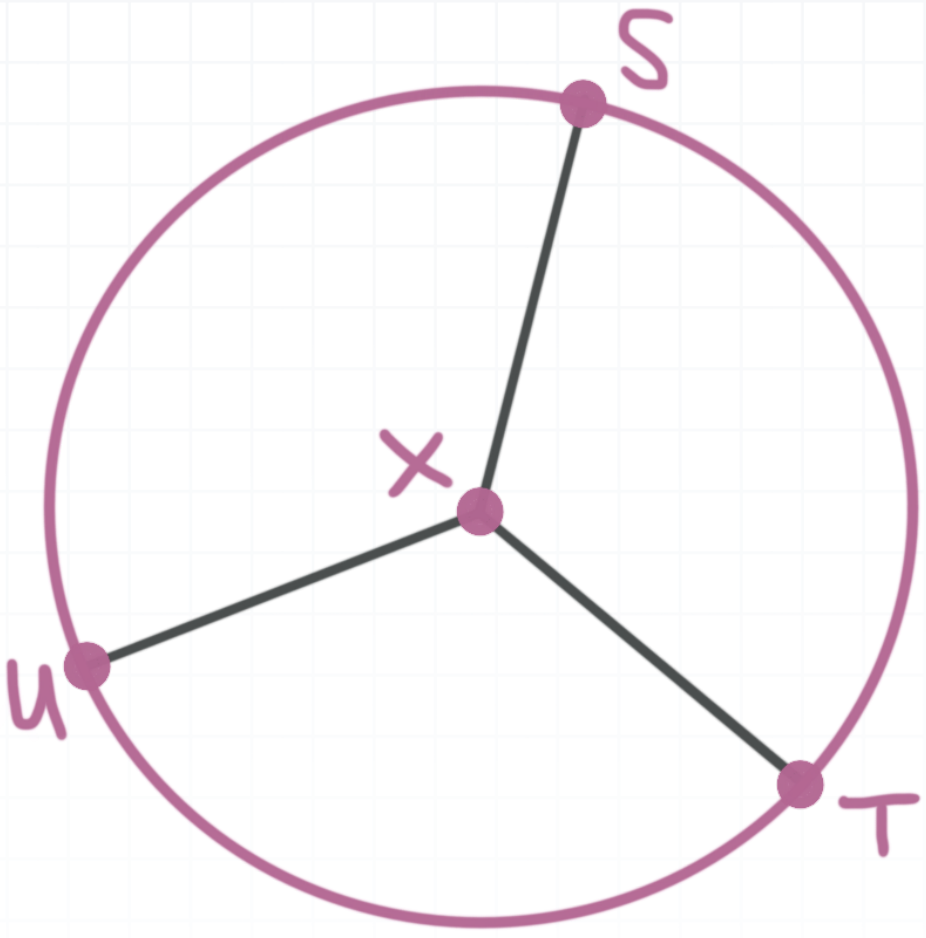
$$x + 90 + x = 180°$$

$$x = 45°$$

Then we know that the measure of arc PQ is $45°$ and the measure of arc QR is $90°$, so the measure of arc PR is $45° + 90° = 135°$.

© Krista King Math

■ 5. In $\odot X$, $\angle UXS \cong \angle SXT \cong \angle UXT$. Find the measure of arc STU.

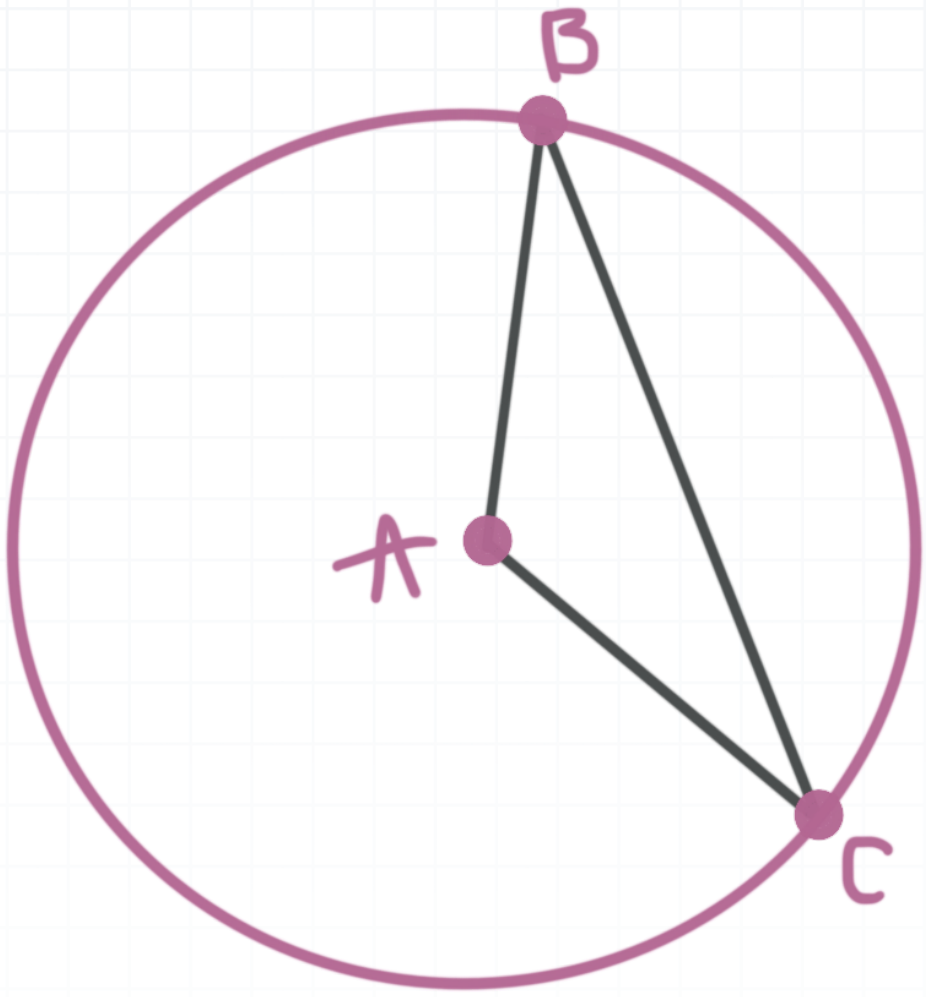


Solution:

$240°$. The measures of arcs ST, TU, and US are congruent, and together they form the full circle, so each of those arcs has measure $360°/3 = 120°$. Therefore, arc STU measures $ST + TU = 120° + 120° = 240°$.

■ 6. In $\odot A$, $m\angle ABC = 15°$. Find the measure of arc BC.

© Krista King Math



Solution:

$150°$. $\triangle ABC$ is isosceles because $\overline{AB}$ and $\overline{AC}$ are radii of $\odot A$.

$$m\angle BAC + m\angle ABC + m\angle BCA = 180°$$

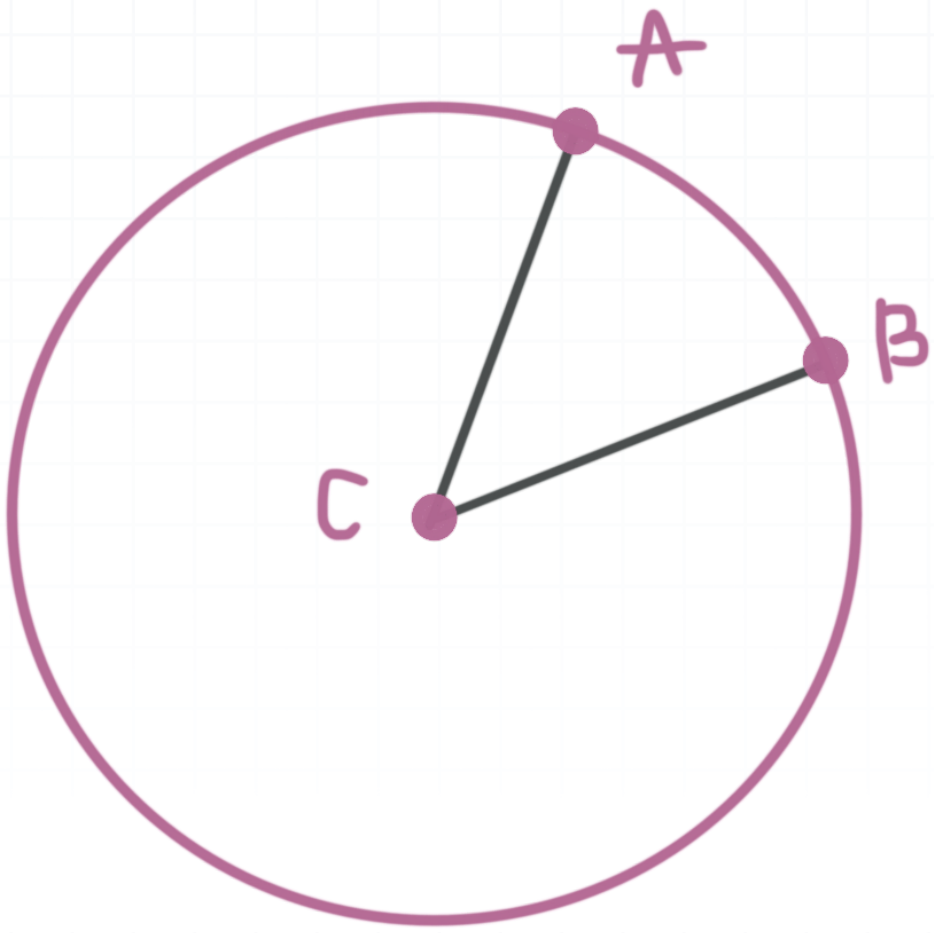
$$m\angle BAC + 15° + 15° = 180°$$

The measure of angle BAC is $m\angle BAC = 150°$, which means the measure of arc BC is also $150°$.

© Krista King Math

ARC LENGTH

■ 1. In $\odot C$, $m\angle ACB = 50°$. Find the length of arc AB if $CA = 14$. Round your answer to the nearest hundredth.



Solution:

12.22°. The circumference of a circle is

$$C = 2\pi r$$

$$C = 2\pi(14)$$

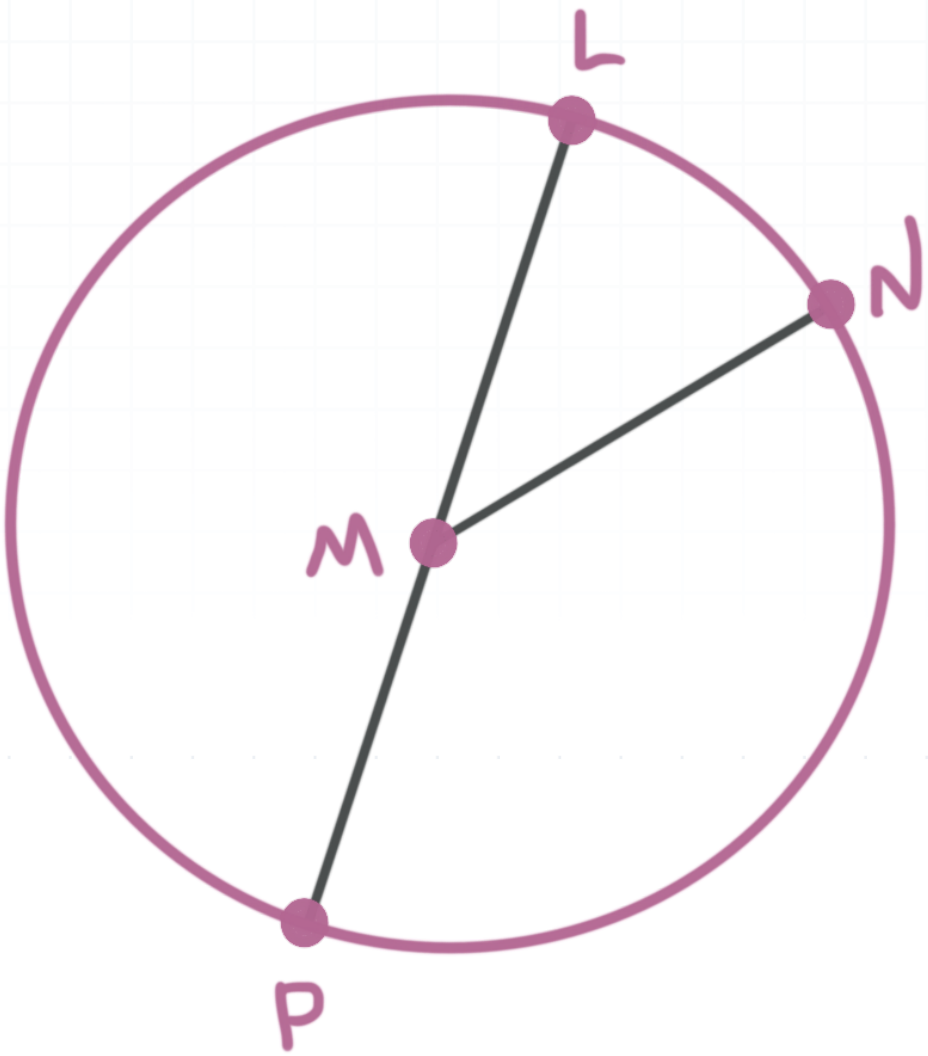
$$C = 87.96$$

© Krista King Math

Then the measure of arc AB is $50°$. The arc length of AB is therefore

$$\frac{50°}{360°}(87.96) = 12.2$$

■ 2. In $\odot M$, $m\angle LMN = 60°$ and $\overline{LP}$ is a diameter. Find the length of arc LPN if $LP = 24$. Round your answer to the nearest hundredth.



Solution:

62.83. The measure of arc LN is $60°$, so the measure of arc LPN is $360° - 60° = 300°$. The circumference is

© Krista King Math

$$C = 2\pi r$$

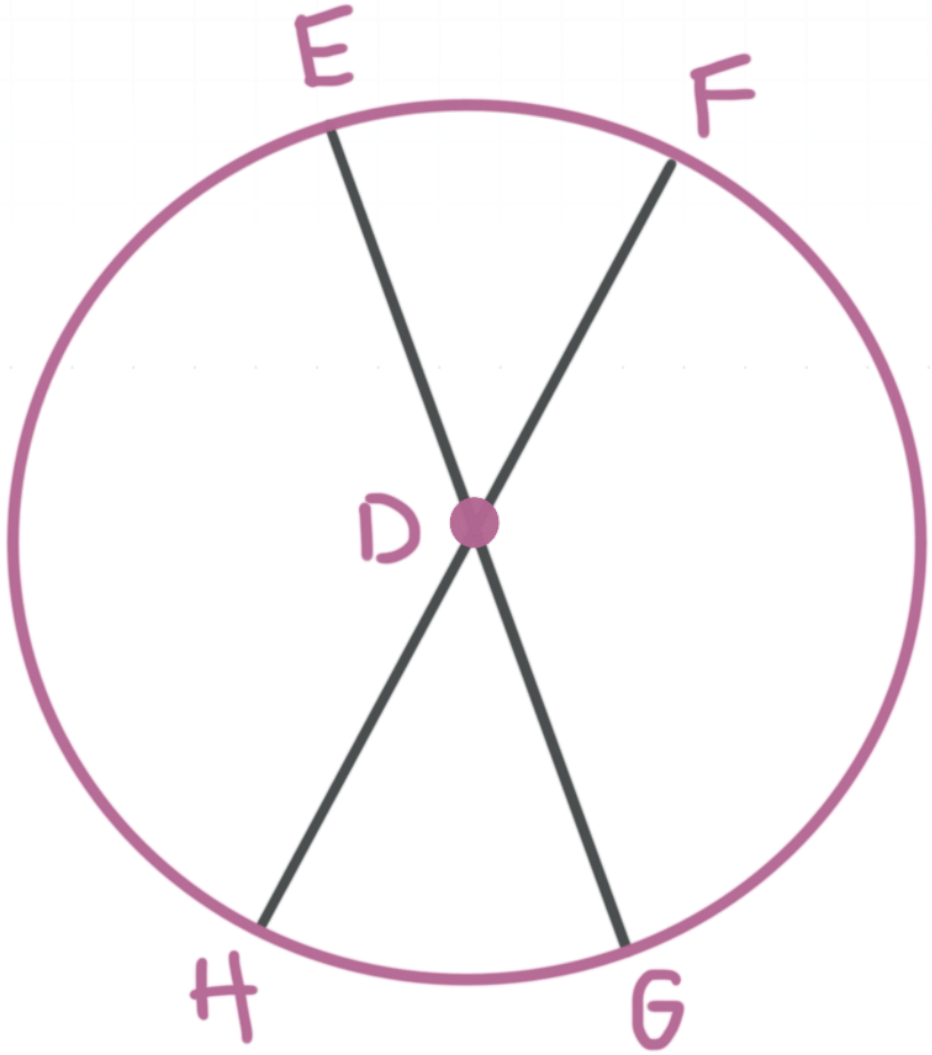
$$C = 24\pi$$

$$C = 75.40$$

So the arc length of LPN is

$$\frac{300^\circ}{360^\circ}(75.40) = 62.83$$

■ 3. $\overline{EG}$ and $\overline{FH}$ are diameters of $\odot D$. Find the length of arc HG if $m\angle EDF = 45^\circ$ and $ED = 16$. Write the exact value.



© Krista King Math

Solution:

4π. The circumference of a circle is

$$C = 2\pi r$$

$$C = 2\pi(16)$$

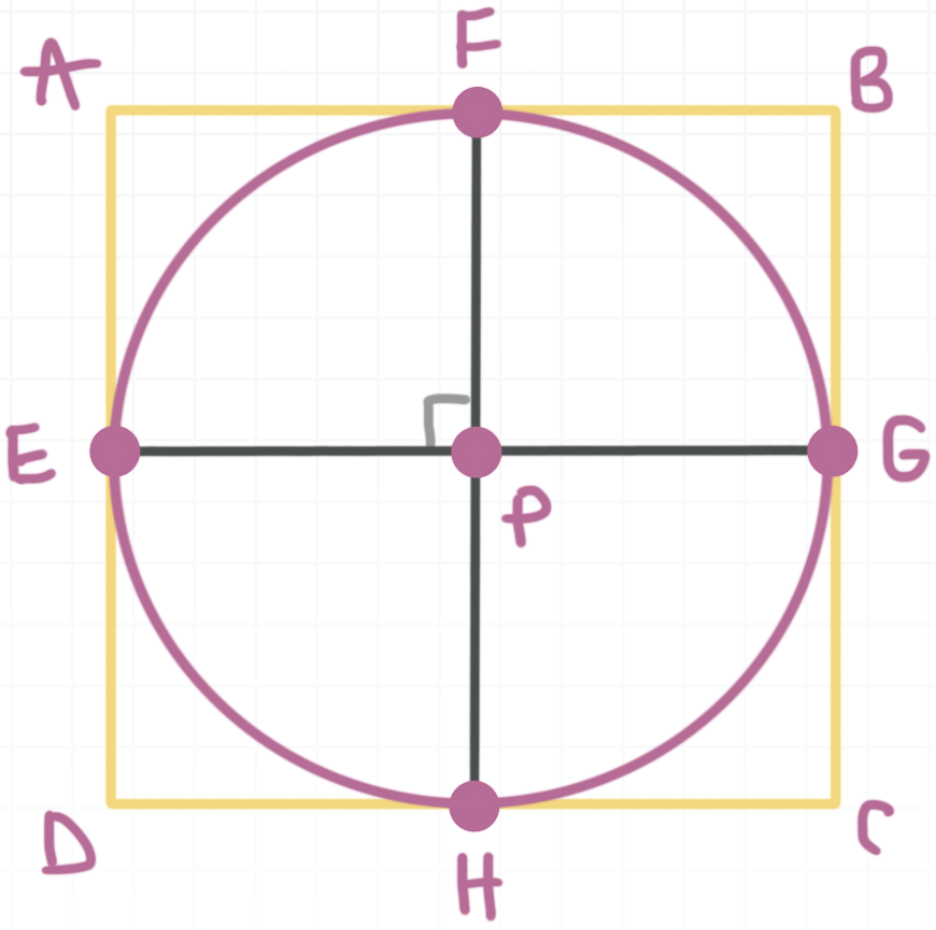
$$C = 32\pi$$

Arcs EF and HG both have a measure of $45°$ since $\angle EDF$ and $\angle HDG$ are vertical angles. Therefore, the length of arc HG is

$$\frac{45°}{360°}(32)\pi = 4\pi$$

■ 4. The area of square $ABCD$ is 144 cm^2 and circle P is inscribed in the square. $\overline{EG}$ and $\overline{FH}$ are perpendicular to one another, and both are diameters of $\odot P$. E, F, G, and H are midpoints of each side of the square. Find the length of arc EF, rounded to the nearest hundredth.

© Krista King Math



Solution:

9.42 cm. The length of each side of the square is $\sqrt{144} = 12$. Because $\odot P$ is inscribed in square $ABCD$, the diameter of the circle is 12. The circumference is

$$C = 2\pi r$$

$$C = 12\pi$$

$$C = 37.70$$

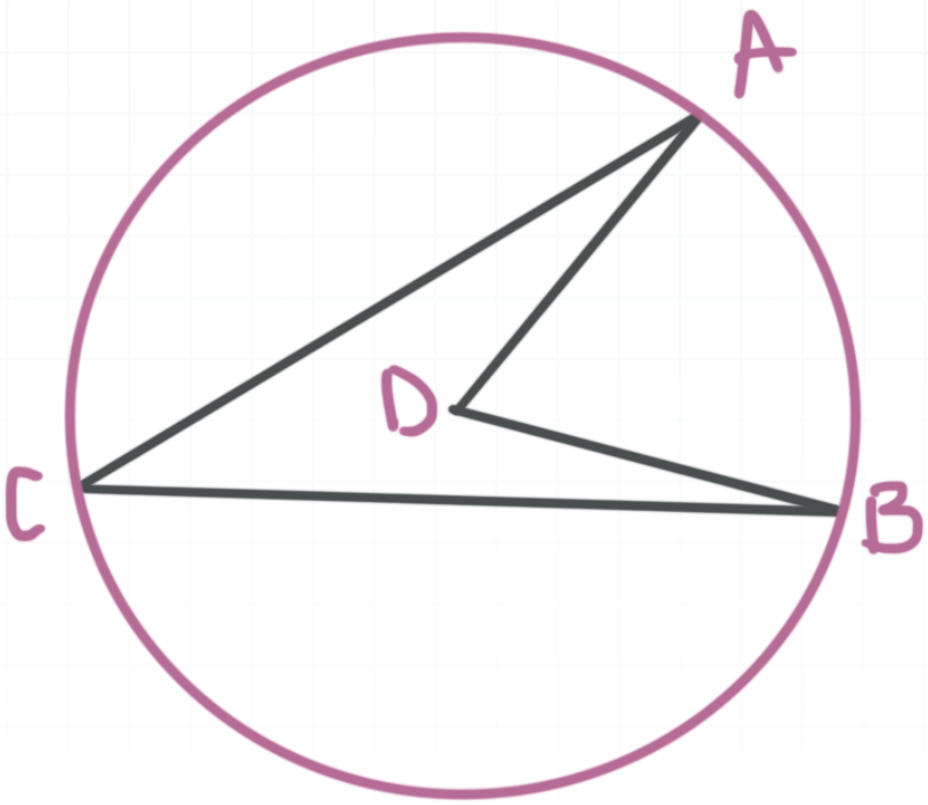
$m\angle EPF = 90°$. So the measure of arc EF is also $90°$. So the length of arc EF is

$$\frac{45°}{360°}(37.70) = 9.42$$

© Krista King Math

INSCRIBED ANGLES OF CIRCLES

■ 1. In $\odot D$, $m\angle ADB = 88°$. Find $m\angle ACB$.



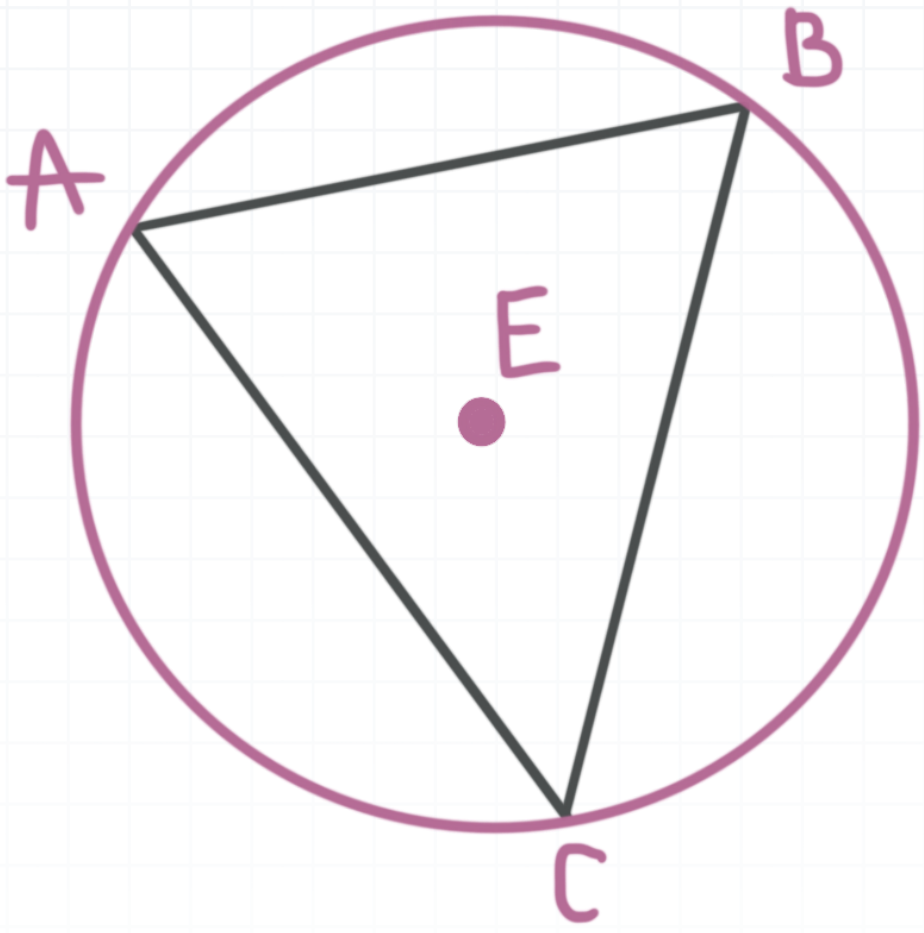
Solution:

$m\angle ACB = 44°$. $m\angle ADB = 88°$, which means the measure of arc AB is $88°$. Which means $m\angle ACB$ is

$$\angle ACB = \frac{1}{2}88° = 44°$$

■ 2. In $\odot E$, $\overline{AC} \cong \overline{CB}$ and $m\angle ABC = 55°$. Find the measure of arc AB.

© Krista King Math



Solution:

$140°$. $\triangle ABC$ is isosceles with $m\angle A = m\angle B = 55°$.

$$m\angle A + m\angle B + m\angle C = 180°$$

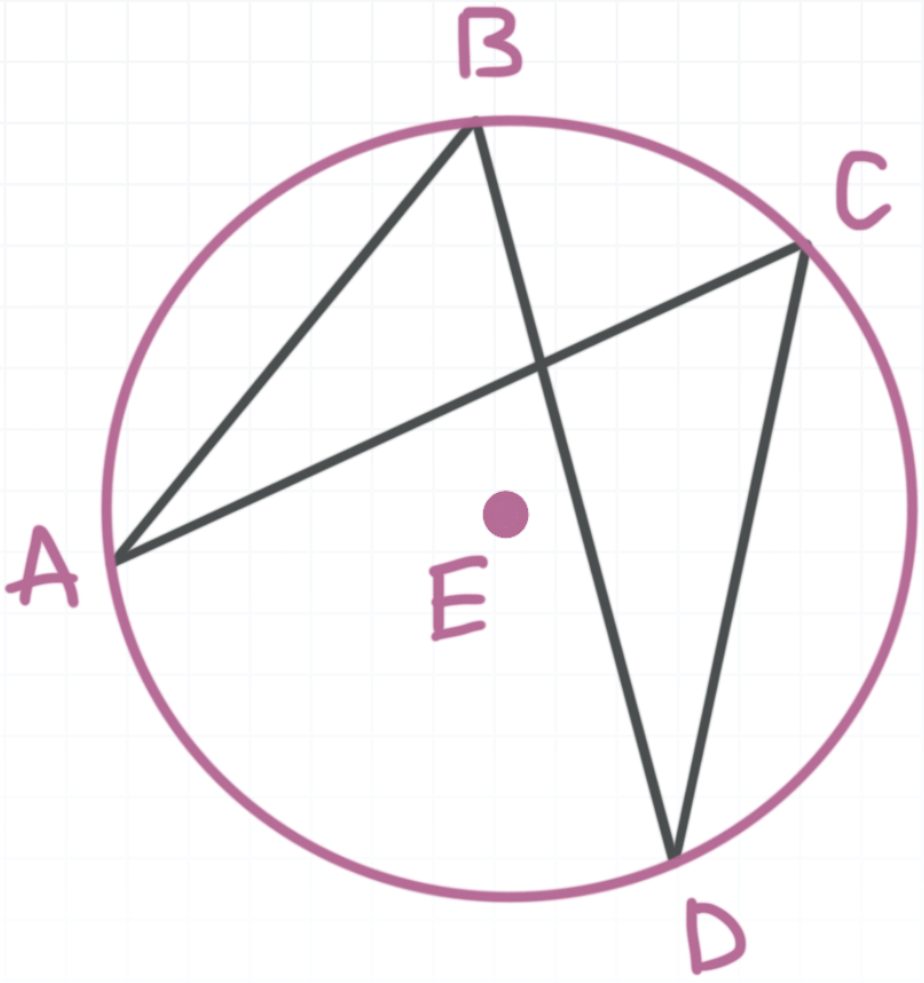
$$55° + 55° + m\angle C = 180°$$

$$m\angle C = 70°$$

Then the measure of arc AB is $2m\angle C = 2(70°) = 140°$.

■ 3. In $\odot E$ the measure of arc AB is $100°$, the measure of arc BC is $40°$, and the measure of CD is $110°$. Find $m\angle ABD$.

© Krista King Math



Solution:

$55°$. Arcs AB, BC, CD, and DA form the full circle, so they sum to $360°$.

$$AB + BC + CD + DA = 360°$$

$$100° + 40° + 110° + DA = 360°$$

$$250° + DA = 360°$$

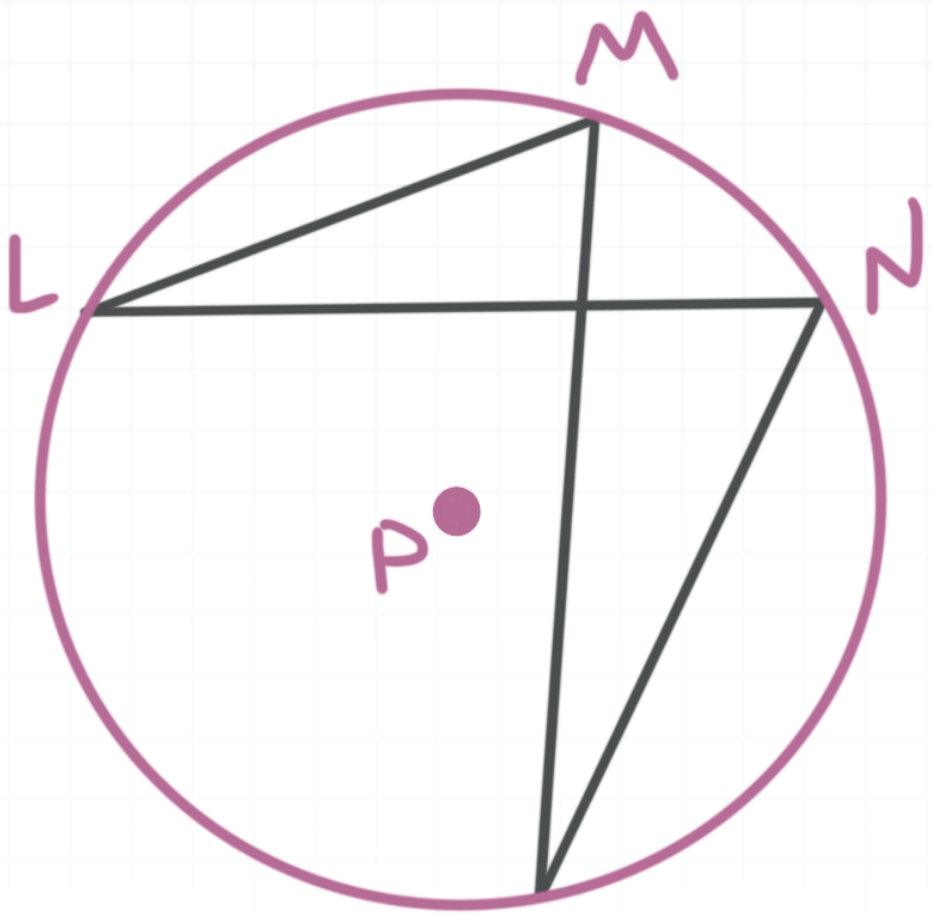
$$DA = 110°$$

Therefore, the measure of angle ABD is

$$m\angle ABD = \frac{1}{2}110° = 55°$$

© Krista King Math

■ 4. In $\odot P$, $m\angle LMO = 2x - 18$ and the measure of arc $LO = 88°$. Find x.



Solution:

$x = 31°$. The measure of angle LMO, if the measure of arc LO is $88°$, is

$$\angle LMO = \frac{1}{2}88°$$

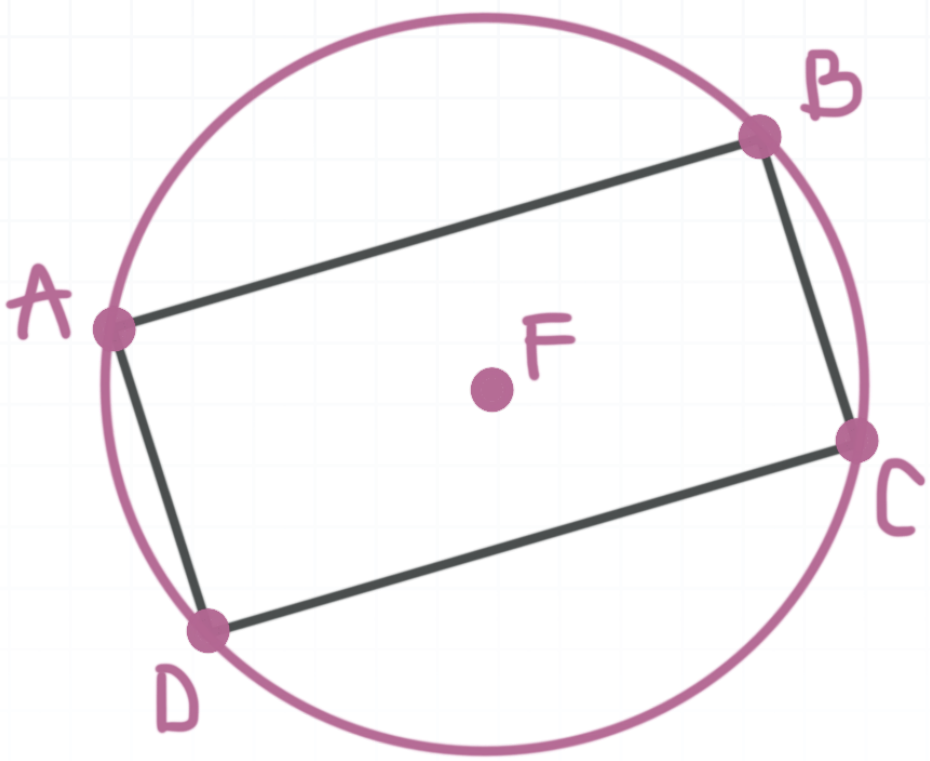
$$2x - 18 = \frac{1}{2}88°$$

$$2x - 18 = 44°$$

© Krista King Math

$x = 31°$

■ 5. Rectangle $ABCD$ is inscribed in $\odot F$ and the measure of arc ACD is $230°$. Find the measure of arc AB.

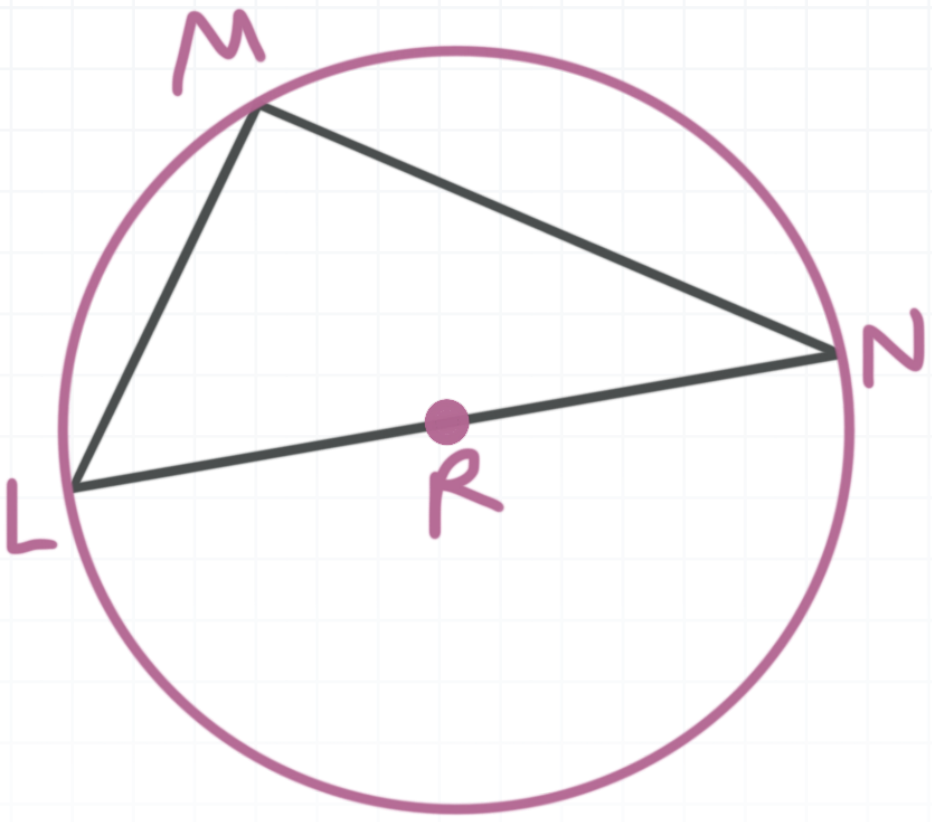


Solution:

$130°$. All of the angles of the rectangle measure $90°$. AC and BD must be diameters of the circle. So the measure of arc DC is $360° - 230° = 130°$. Arcs AB and DC are congruent, so they both have a measure of $130°$.

■ 6. In $\odot R$, $\overline{LN}$ is a diameter, $m\angle MLN = 4x + 20$, and $m\angle LNM = 5x - 38$. Find the measure of arc LM.

© Krista King Math



Solution:

$44°$. We know that

$$m\angle LMN = \frac{1}{2}(180°) = 90°$$

And since the three interior angles of a triangle always sum to $180°$,

$$m\angle LMN + m\angle LNM + m\angle MLN = 180°$$

$$90° + 4x + 20 + 5x - 38 = 180°$$

$$9x + 72 = 180°$$

$$x = 12$$

Therefore, $m\angle LNM = 5(12) - 38 = 22°$, so

© Krista King Math

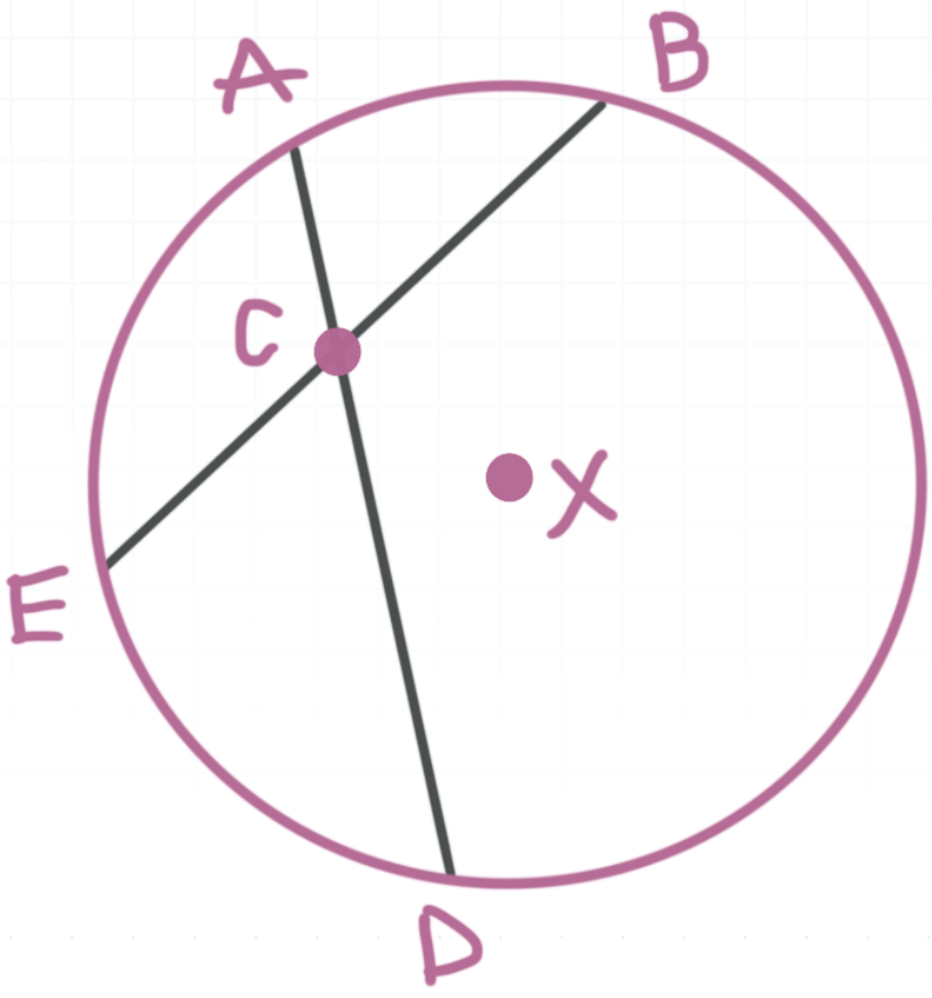
$$22^\circ = \frac{1}{2}(\text{arc } LM)$$

$$LM = 44^\circ$$

© Krista King Math

VERTEX ON, INSIDE AND OUTSIDE THE CIRCLE

■ 1. $\overline{AD}$ and $\overline{EB}$ are chords of $\odot X$. The measure of arc AB is $35°$ and the measure of arc ED is $85°$. Find $m\angle ECD$.



Solution:

$60°$. $m\angle ECD$ is given by

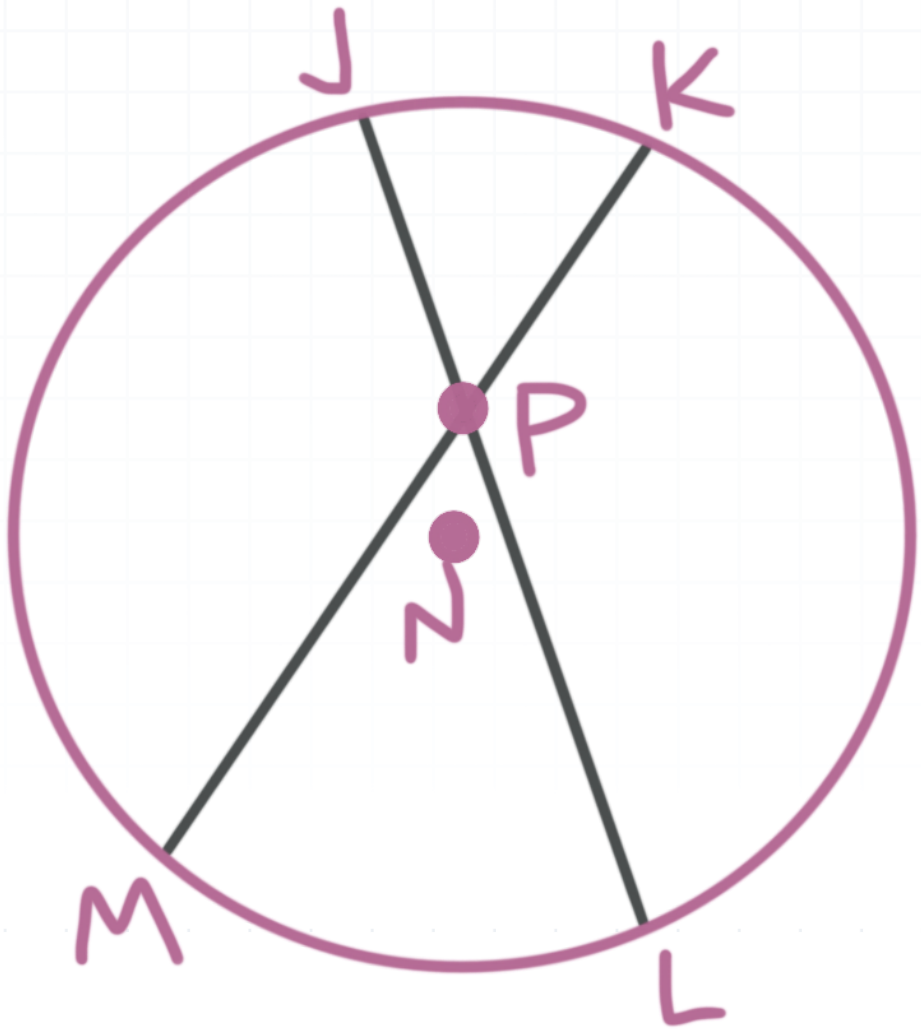
$$m\angle ECD = \frac{1}{2}(\text{arc } ED + \text{arc } AB)$$

$$m\angle ECD = \frac{1}{2}(85° + 35°)$$

© Krista King Math

$m\angle ECD = 60°$

■ 2. $\overline{JL}$ and $\overline{KM}$ are chords of $\odot N$. The measure of arc JK is $25°$ and $m\angle JPK = 40°$. Find the measure of arc ML.



Solution:

$55°$. The measure of $\angle JPK$ is given by

$$m\angle JPK = \frac{1}{2}(\text{arc } ML + \text{arc } JK)$$

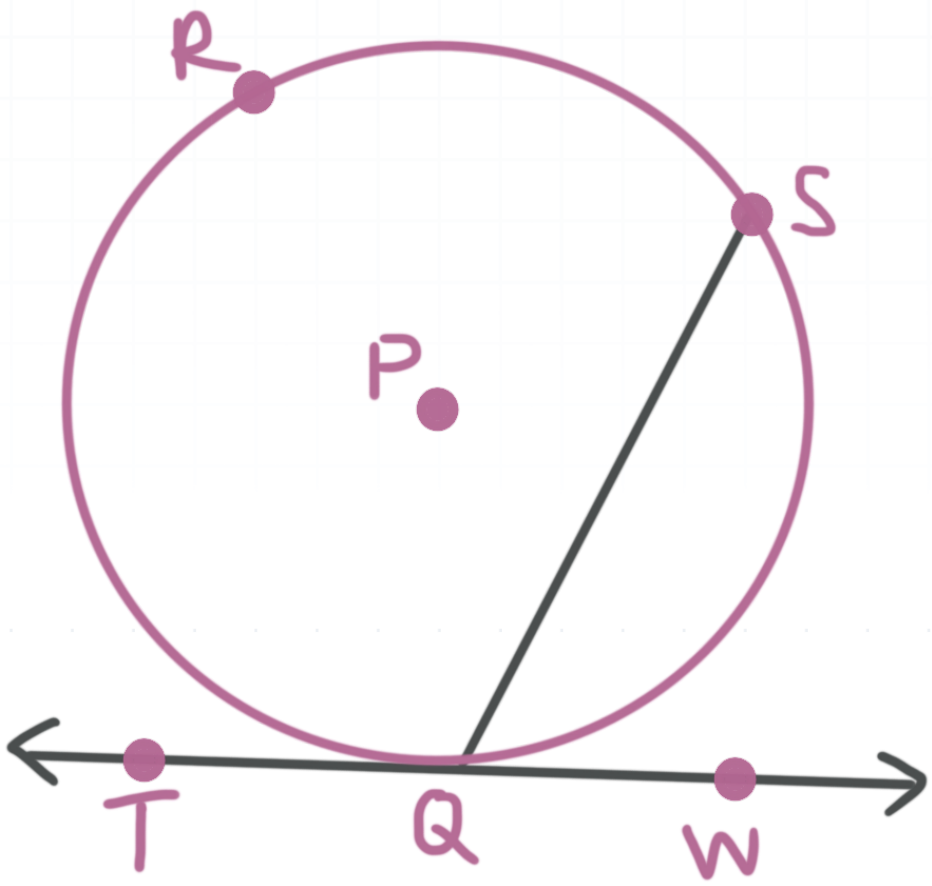
© Krista King Math

$$40° = \frac{1}{2}(\text{arc } ML + 25°)$$

$$80° = \text{arc } ML + 25°$$

$$\text{arc } ML = 55°$$

■ 3. $\overline{SQ}$ is a chord and $\overline{TW}$ is a tangent line of $\odot P$. The measure of arc SRQ is $194°$. Find $m\angle SQW$.



Solution:

$83°$. We can find $m\angle SQW$ as

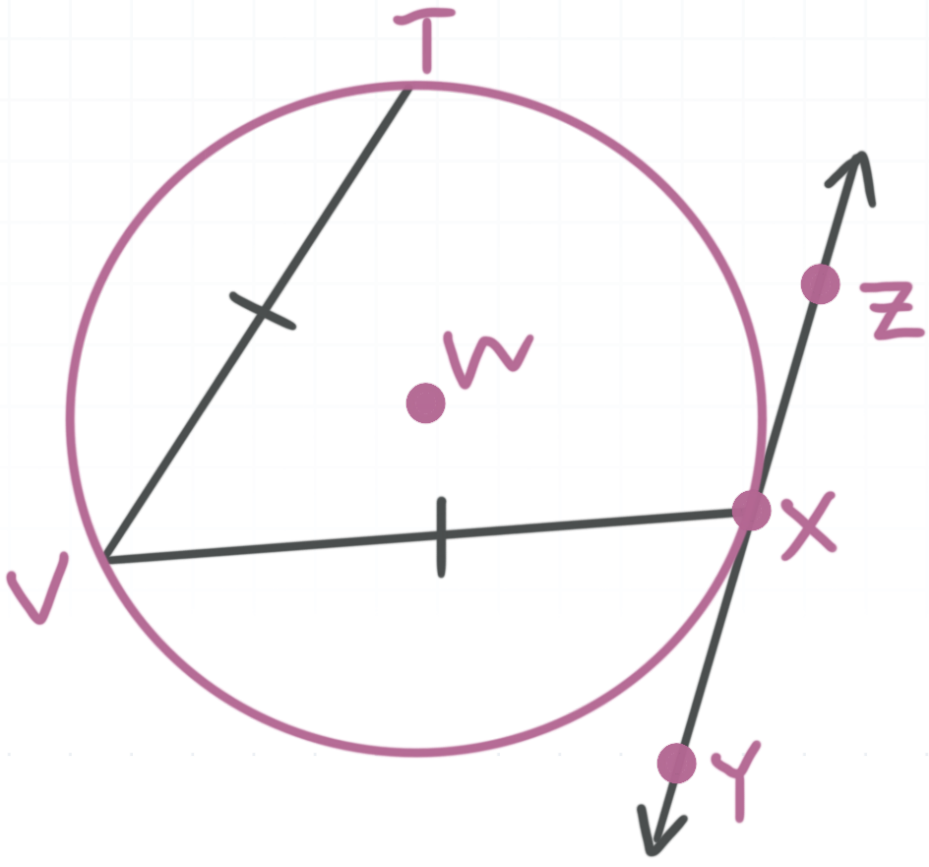
$$m\angle SQW = \frac{1}{2}(\text{arc } SW)$$

© Krista King Math

$$\text{arc } SW = 360° - \text{arc } SRQ = 360° - 194° = 166°$$

$$m\angle SQW = \frac{1}{2}(166°) = 83°$$

■ 4. $\overline{TV}$ and $\overline{VX}$ are congruent chords, and $\overline{ZY}$ is a tangent line of $\odot W$. If $m\angle TVX = 48°$, find $m\angle VXY$.



Solution:

$66°$. Find the length of different arcs in the circle.

$$\text{arc } TX = 2(m\angle TVX) = 2(48) = 96$$

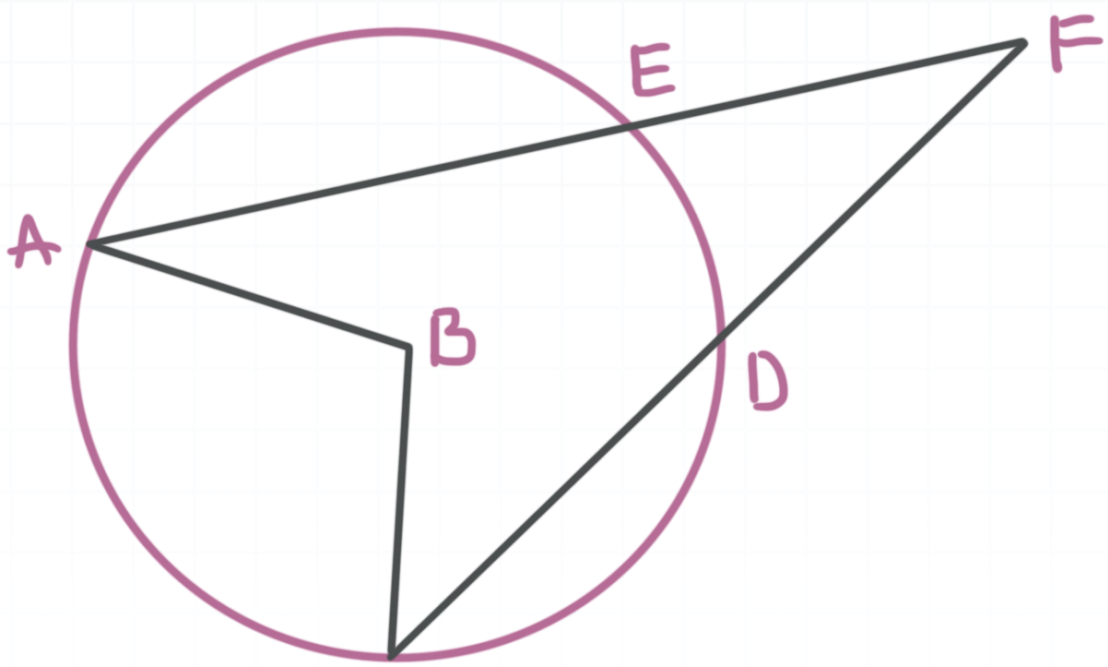
$$\text{arc } VX = \frac{1}{2}(360° - \text{arc } TX) = \frac{1}{2}(360° - 96°) = 132°$$

© Krista King Math

Then $m\angle VXY$ is given by

$$m\angle VXY = \frac{1}{2}(132^\circ) = 66^\circ$$

■ 5. arc $AC = 98^\circ$ and arc $ED = 54^\circ$ in $\odot B$. Find $m\angle AFC$.



Solution:

22°. We can find $m\angle AFC$ as

$$m\angle AFC = \frac{1}{2}(\text{arc } AC - \text{arc } ED)$$

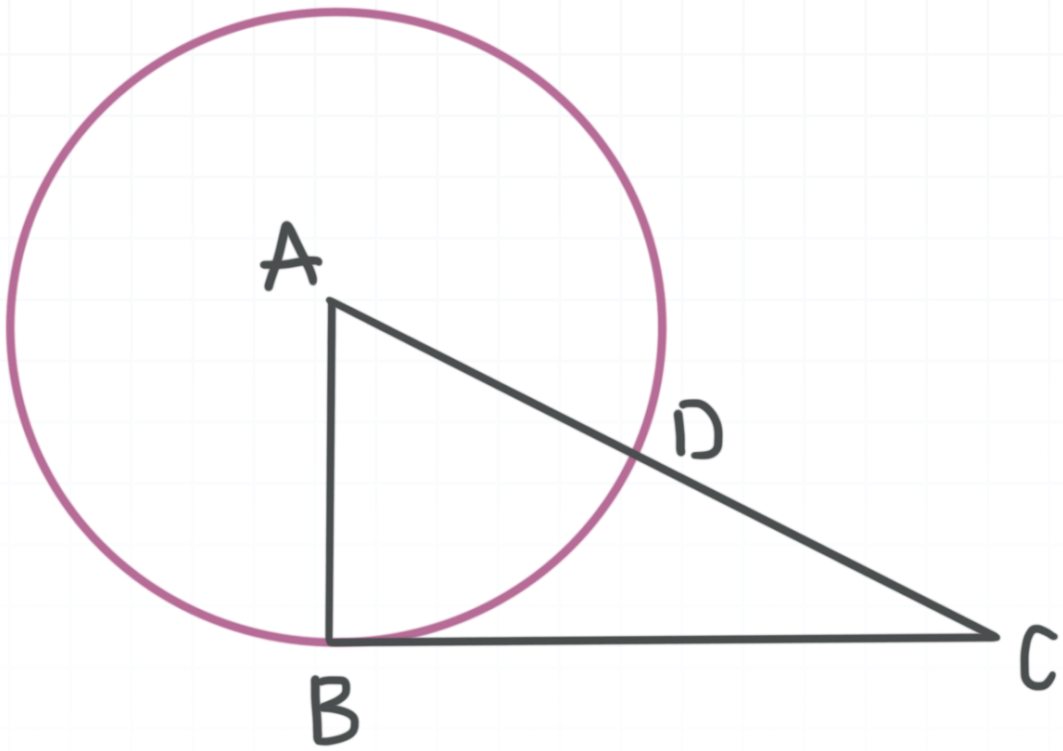
$$m\angle AFC = \frac{1}{2}(98^\circ - 54^\circ)$$

$$m\angle AFC = 22^\circ$$

© Krista King Math

TANGENT LINES OF CIRCLES

■ 1. $\odot A$ has radius AB and tangent line $\overline{BC}$. If $AB = 6$ and $BC = 8$, find DC.



Solution:

$DC = 4$. We can plug some lengths from the triangle into the Pythagorean Theorem.

$$AB^2 + BC^2 = AC^2$$

$$6^2 + 8^2 = AC^2$$

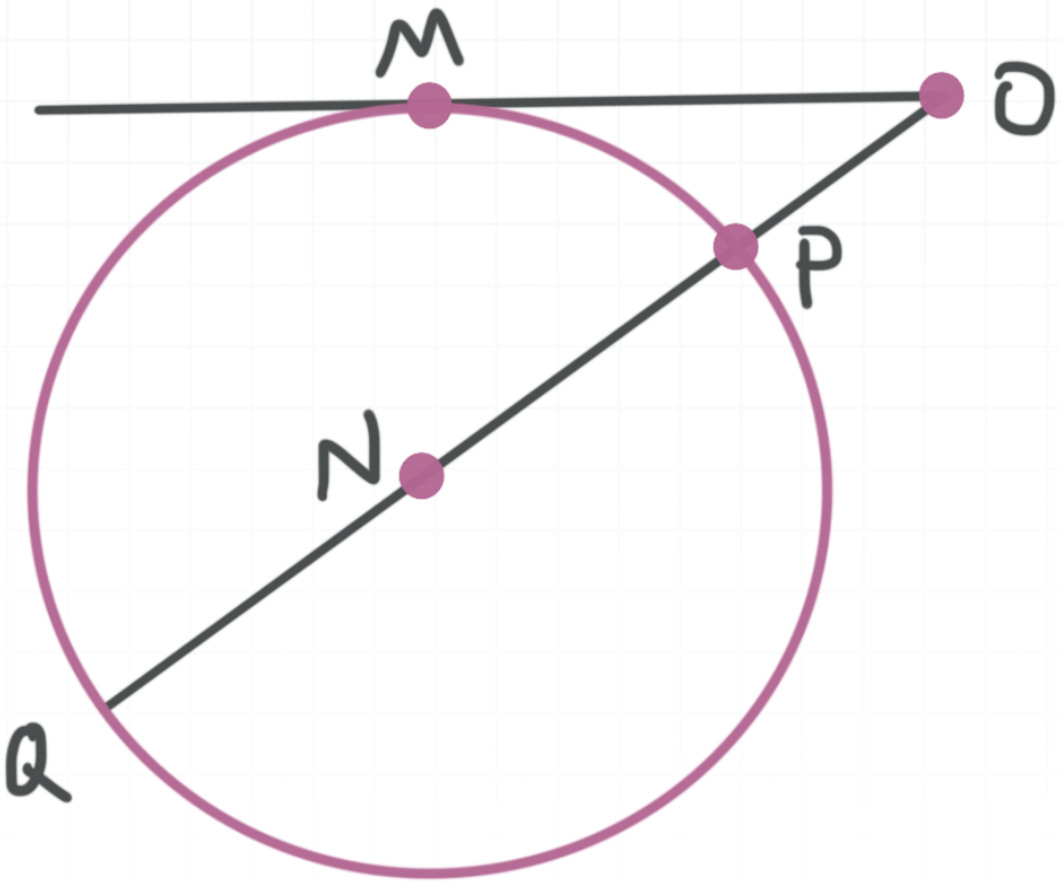
$$100 = AC^2$$

$$10 = AC$$

Then $DC = 10 - AD = 10 - 6 = 4$.

© Krista King Math

■ 2. $\overline{MO}$ is a tangent line of $\odot N$. If $MO = 12$ and $PO = 8$, find the length of the radius.



Solution:

5. Let the length of the radius be x. Then use the Pythagorean Theorem.

$$MN^2 + MO^2 = ON^2$$

$$x^2 + 12^2 = (x + 8)^2$$

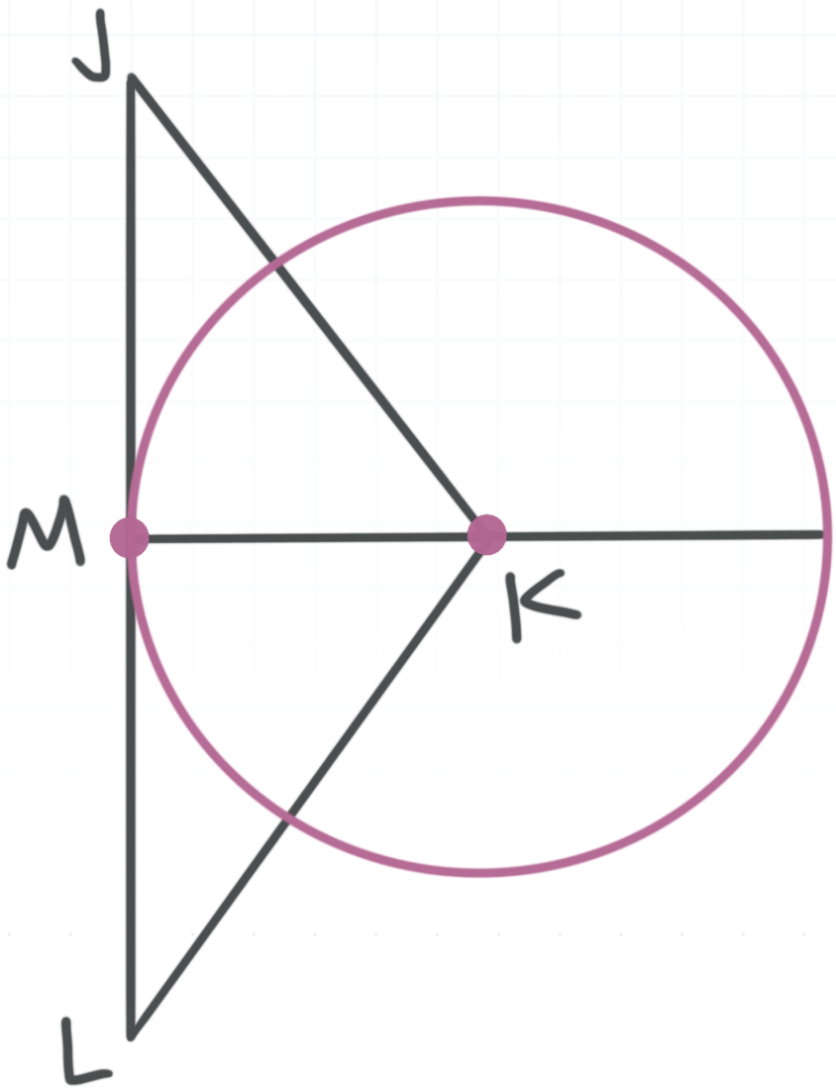
$$x^2 + 144 = x^2 + 16x + 64$$

$$16x = 80$$

© Krista King Math

$x = 5$

■ 3. $\triangle JKL$ is isosceles, $\overline{JL}$ is a tangent line, $JM = LM$ and $m\angle JKL = 120°$. If $MK = 8$, find JL.



Solution:

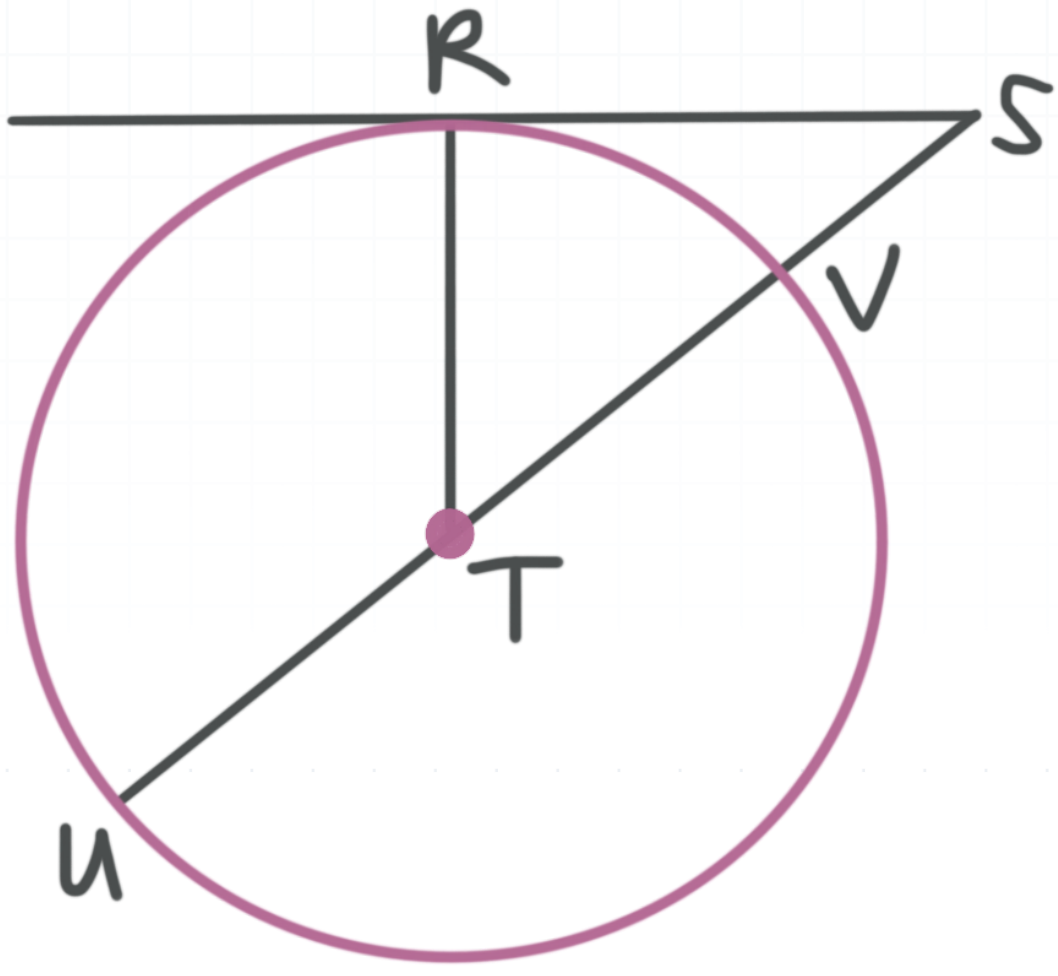
$16\sqrt{3}$. The triangles $\triangle JMK$ and $\triangle LMK$ are congruent and $\overline{MK}$ bisects $\angle JKL$. We know $m\angle JKL = 120°$, so $m\angle JKM = m\angle LKM = 60°$. $m\angle KMJ$ must be $90°$ because $\overline{MK}$ is a radius and $\overline{JL}$ is a tangent line. Use the rules for $30 - 60 - 90$ triangles to find JM.

© Krista King Math

$$JM = MK\sqrt{3} = 8\sqrt{3}$$

$$JL = 2JM = 2\left(8\sqrt{3}\right) = 16\sqrt{3}$$

■ 4. In $\odot T$, $\overline{RS}$ is a tangent line and the diameter $\overline{UV}$ has length of 6. Find VS if $RS = 4$.



Solution:

$VS = 2$. We know that $TV = RT = 3$ because they are both radii and are half the length of the diameter. From the Pythagorean Theorem,

$$RT^2 + RS^2 = TS^2$$

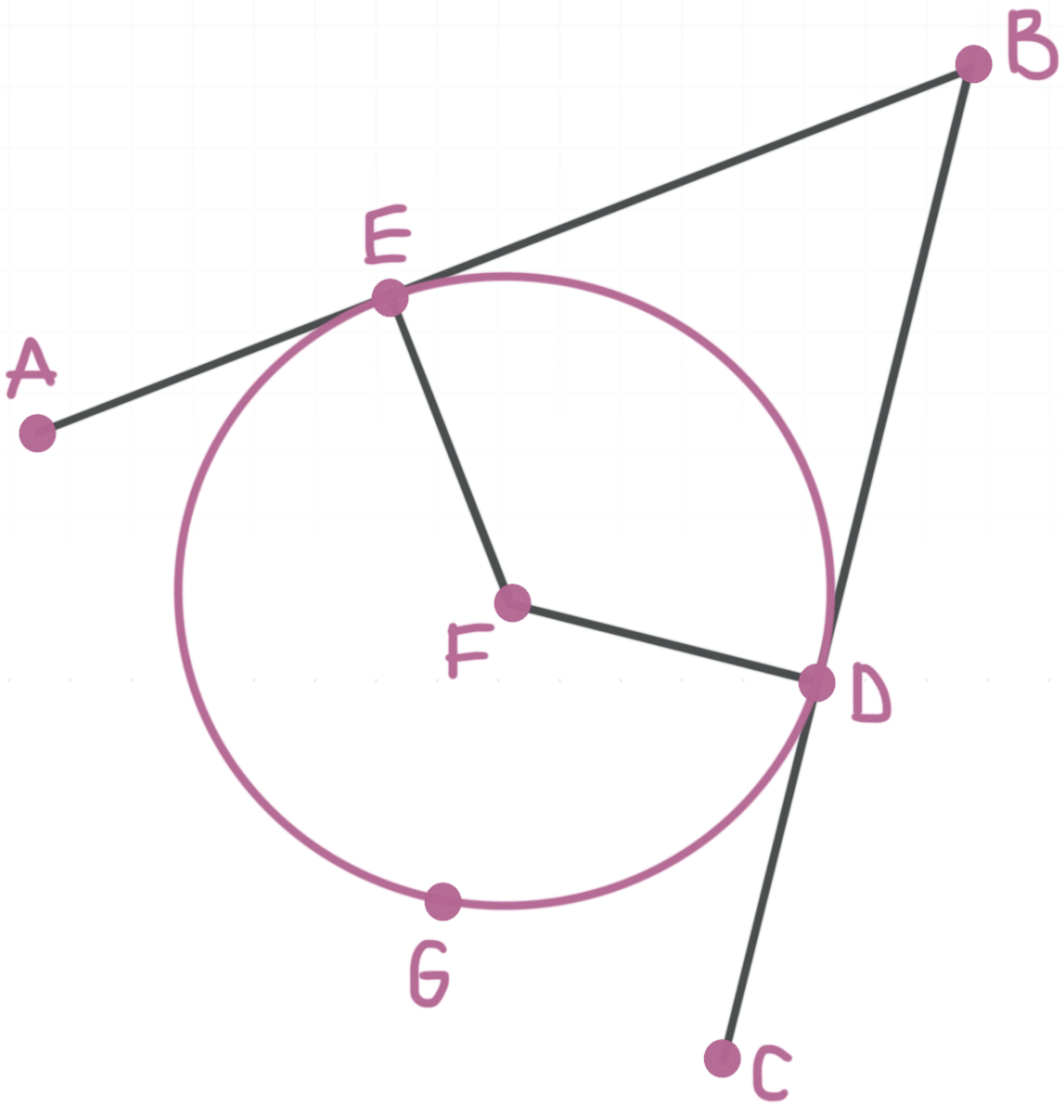
© Krista King Math

$$3^2 + 4^2 = TS^2$$

$$TS = 5$$

Which means the length of VS is $VS = TS - TV = 5 - 3 = 2$.

■ 5. arc $EGC = 240°$ and $\overline{BF}$ bisects $\angle EFD$. Find the length of the radius of $\odot F$ if $FB = 14$.



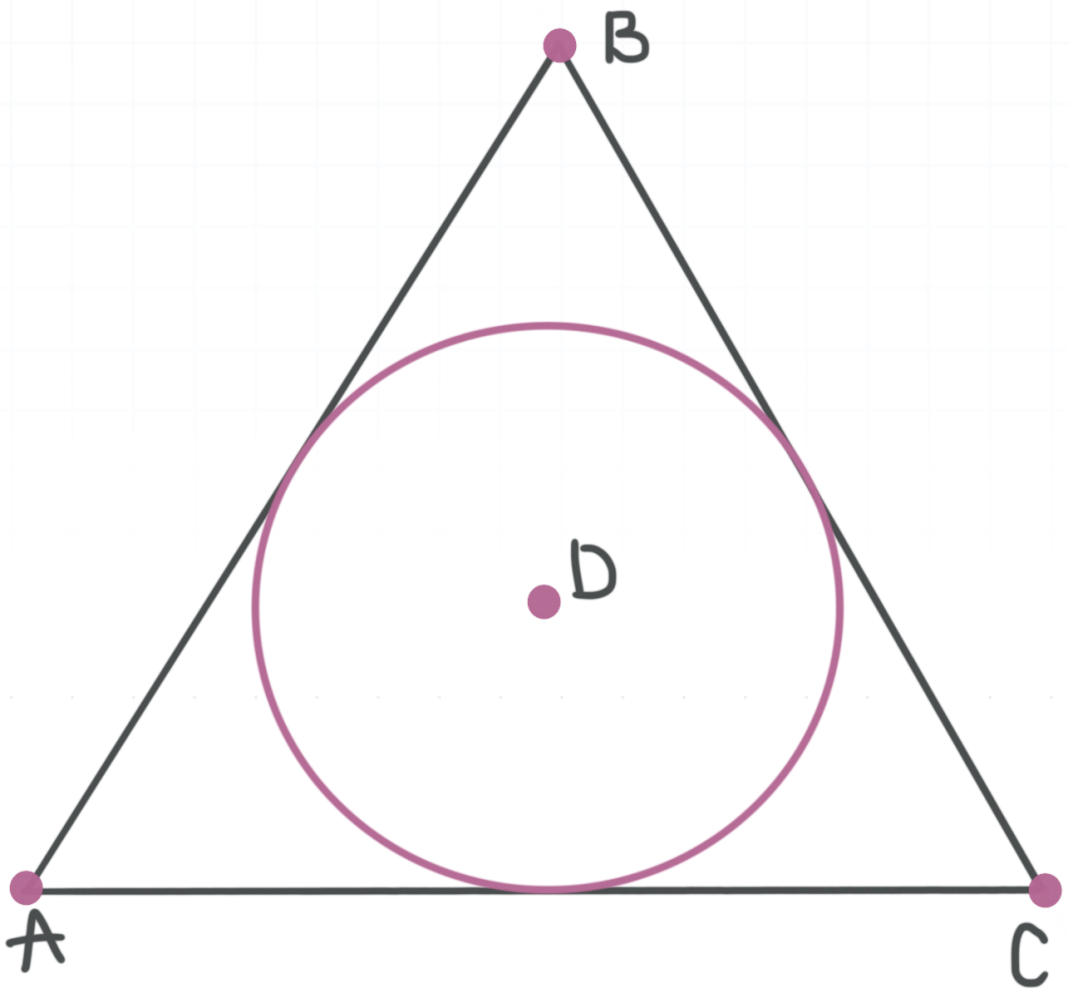
Solution:

7. We know that $m\angle EFD = 360° - 240° = 120°$.

© Krista King Math

Draw in $\overline{FB}$ and note $\triangle FEB$ and $\triangle DFB$ are congruent $30-60-90$ triangles. $\overline{FB}$ is a hypotenuse of these triangles and has a measure of 14. $\overline{EF}$ and $\overline{FD}$ are the shortest legs of these triangles and have a measure of $14/2 = 7$.

■ 6. Find the perimeter of $\triangle ABC$ if the radius of $\odot D$ is 10 feet and $\triangle ABC$ is equilateral.



Solution:

$60\sqrt{3}$. We know that $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are congruent since $\triangle ABC$ is equilateral. Which means $\angle A = \angle B = \angle C = 60°$.

© Krista King Math

Since the diameter is 10, the length of half of each side must be $10\sqrt{3}$ if you draw segments at $\overline{AD}$, $\overline{DB}$, and $\overline{DC}$. Each side must have a length of

$$2\left(10\sqrt{3}\right) = 20\sqrt{3}$$

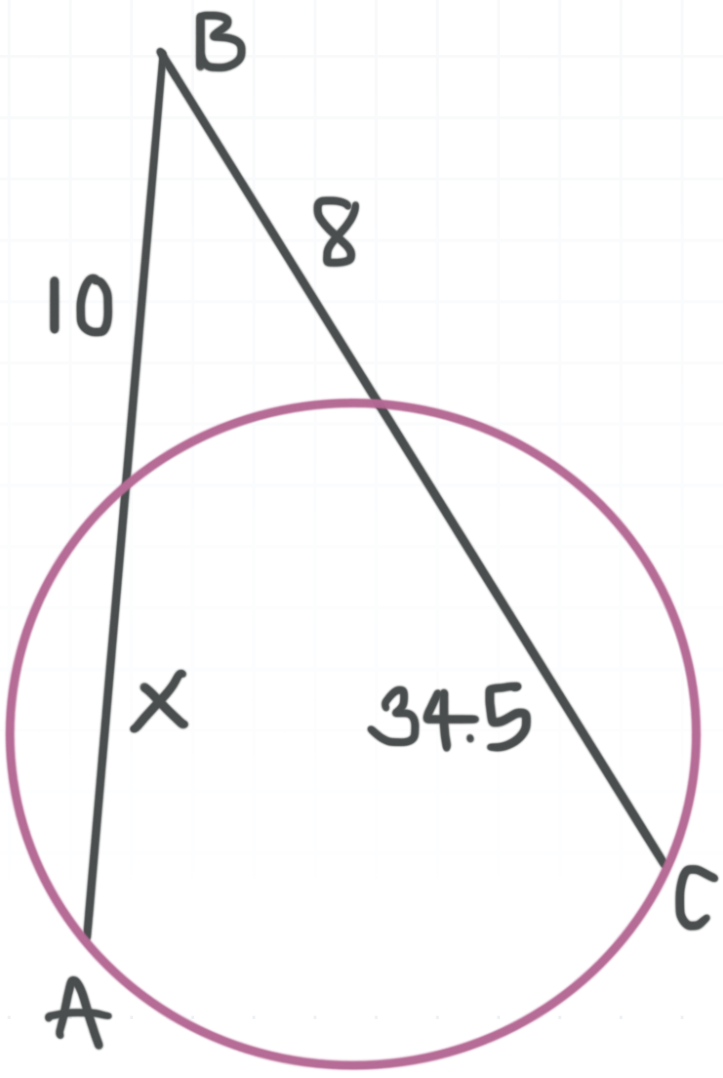
Find the perimeter by adding up the three sides to get

$$P = 20\sqrt{3} + 20\sqrt{3} + 20\sqrt{3} = 60\sqrt{3}$$

© Krista King Math

INTERSECTING TANGENTS AND SECANTS

■ 1. $\overline{AB}$ and $\overline{CB}$ are secants and intersect at B. Find the value of x.



Solution:

$x = 24$. From the figure, we know that

$$10(10 + x) = 8(8 + 34.5)$$

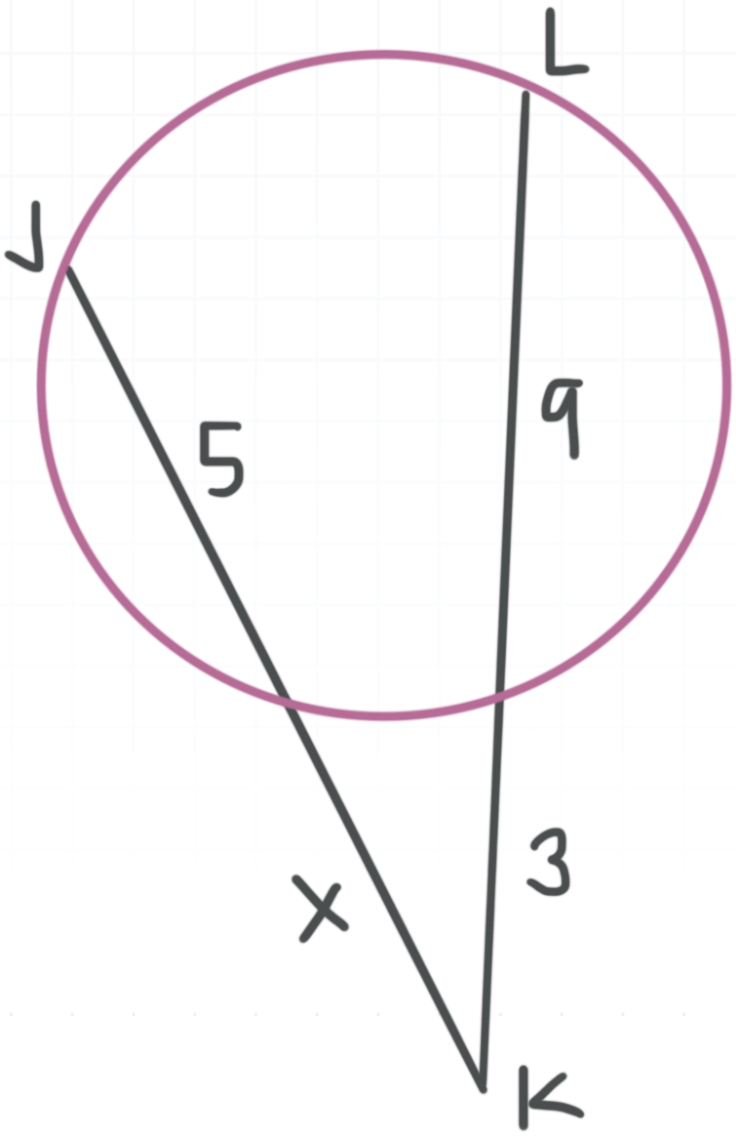
$$100 + 10x = 340$$

$$10x = 240$$

© Krista King Math

$x = 24$

■ 2. $\overline{JK}$ and $\overline{LK}$ are secants and intersect at K. Find the value of x.



Solution:

$x = 4$. From the figure, we know that

$$x(x + 5) = 3(3 + 9)$$

$$x^2 + 5x = 36$$

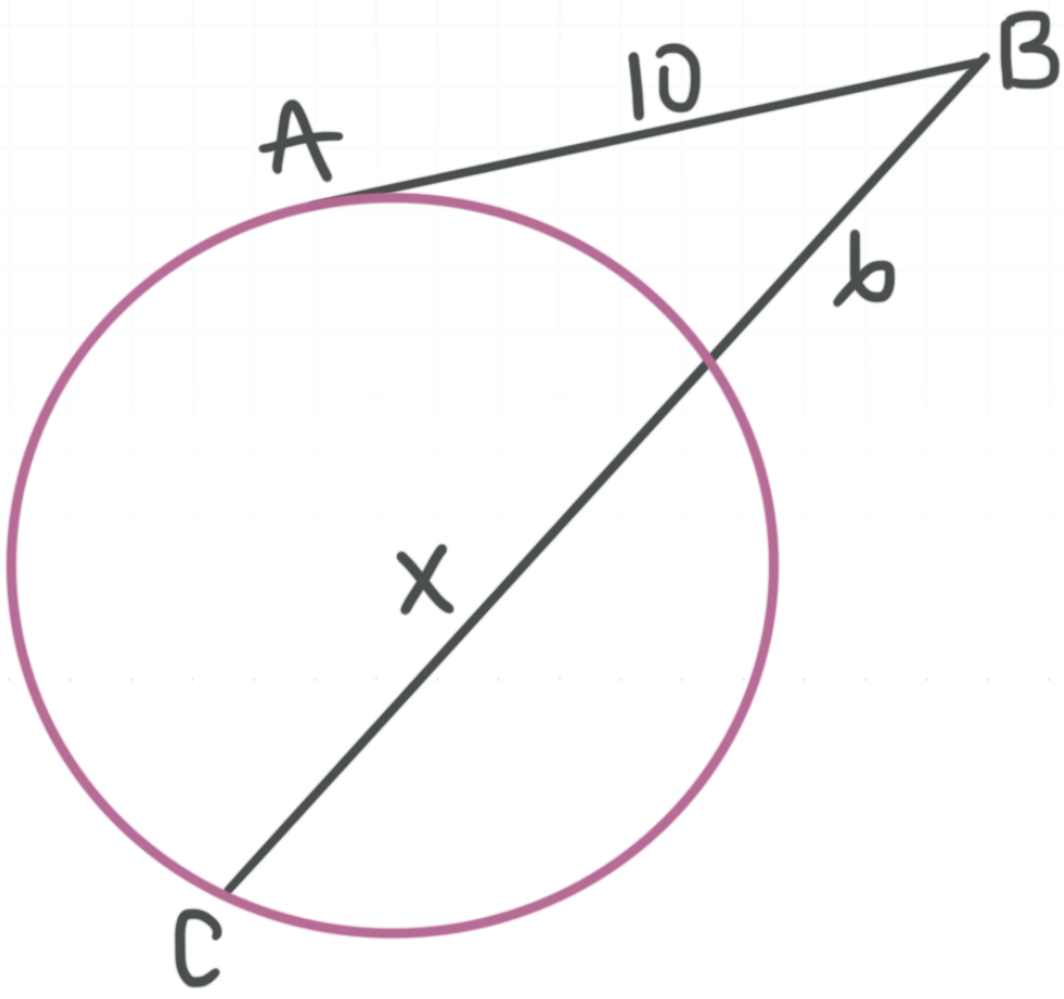
© Krista King Math

$$x^2 + 5x - 36 = 0$$

$$(x + 9)(x - 4) = 0$$

Either $x = -9$ or $x = 4$, but x represents a physical distance, which means it can't have a negative value, so $x = 4$.

■ 3. $\overline{AB}$ is a tangent line and $\overline{BC}$ is a secant of the circle. Find the value of x.



Solution:

$x = 32/3$. From the figure, we know that

$$10^2 = 6(6 + x)$$

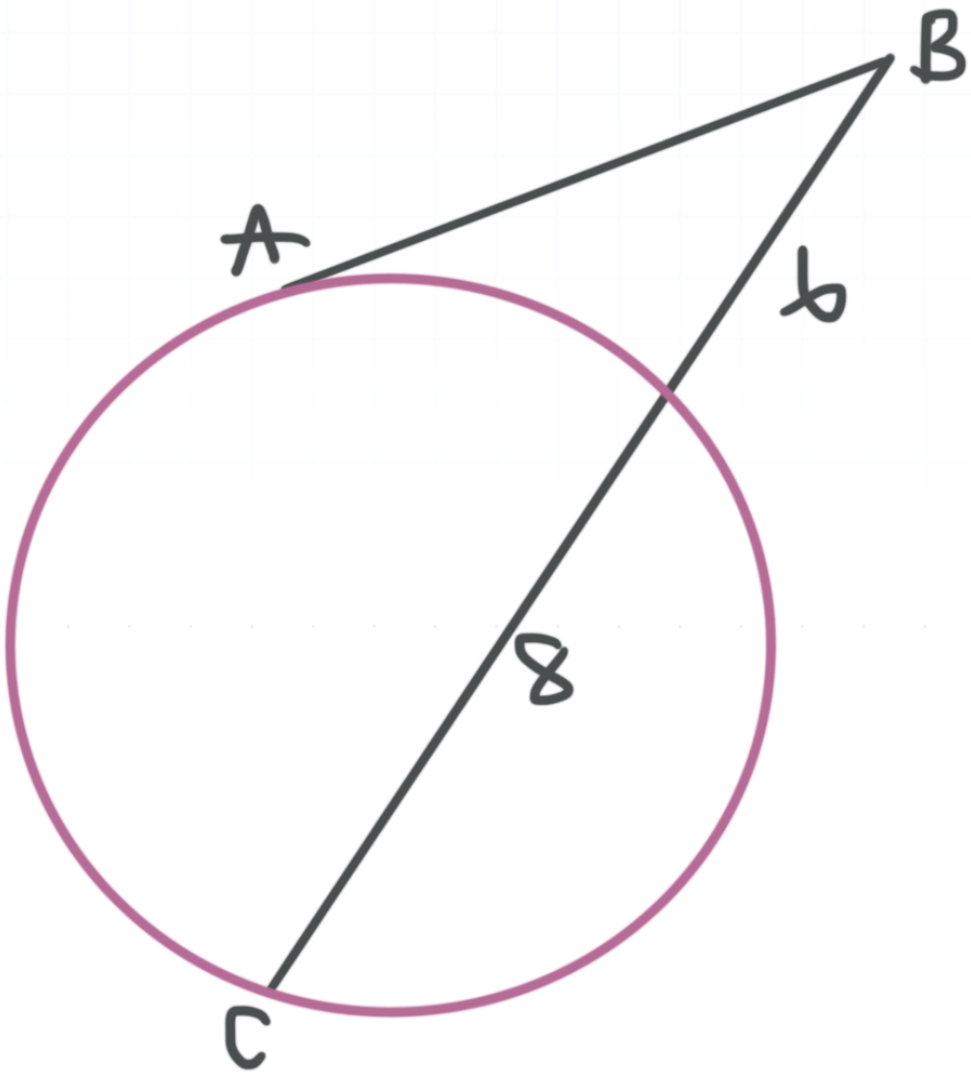
© Krista King Math

$$100 = 36 + 6x$$

$$6x = 64$$

$$x = \frac{32}{3}$$

■ 4. $\overline{AB}$ is a tangent line and $\overline{CB}$ is a secant of the circle. Find the length of AB.



Solution:

$2\sqrt{21}$. From the figure, the length of AB is

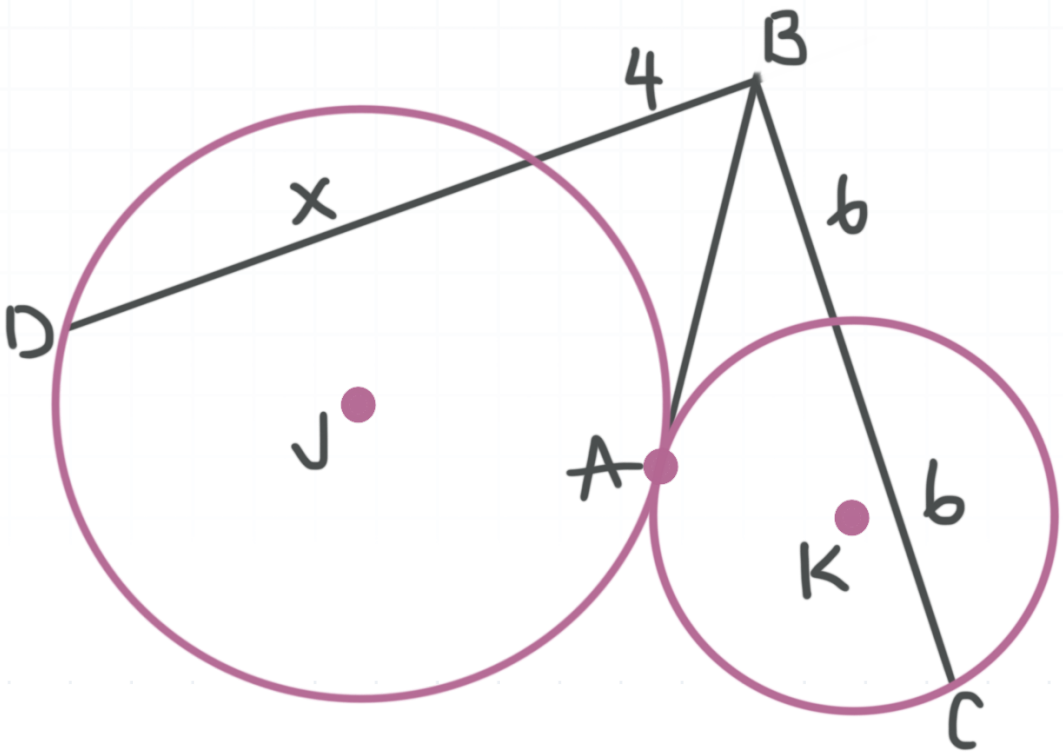
© Krista King Math

$$x^2 = 6(6+8)$$

$$x^2 = 84$$

$$x = \sqrt{84} = 2\sqrt{21}$$

■ 5. $\overline{DB}$ is a secant of $\odot J$ and $\overline{CB}$ is a secant of $\odot K$. $\overline{AB}$ is a tangent for both circles. Find x.



Solution:

14. From the figure, we know that

$$AB^2 = 6(6+6) = 72$$

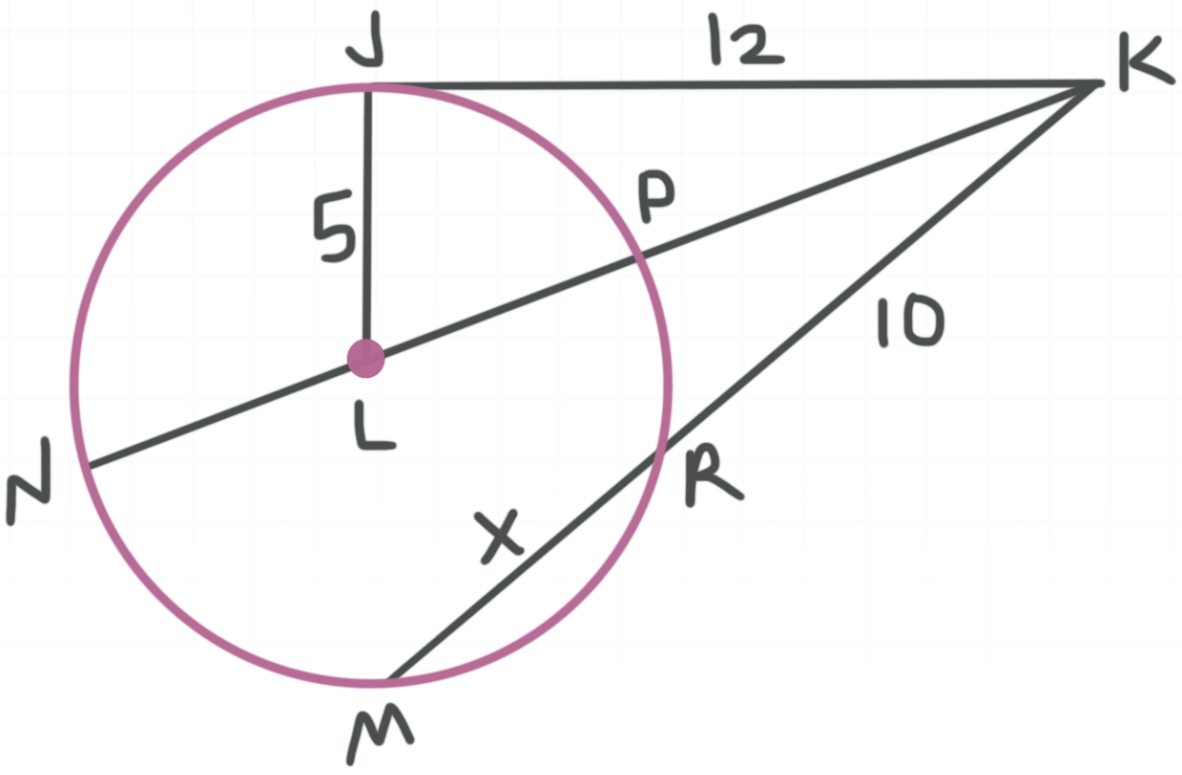
$$AB^2 = 4(4+x)$$

© Krista King Math

$$72 = 4(4 + x)$$

$$x = 14$$

■ 6. $\overline{JK}$ is a tangent line, $\overline{KN}$ and $\overline{KM}$ are secants, and $\overline{LJ}$ and $\overline{LP}$ are radii of $\odot L$. Find x.



Solution:

$x = 4.4$. We can use the Pythagorean Theorem.

$$LK^2 = JK^2 + LJ^2$$

$$LK^2 = 12^2 + 5^2 = 169$$

$$LK = 13$$

© Krista King Math

Therefore, $PK = 8$ and $NP = 10$. Which means that we can set up an equation to solve for x.

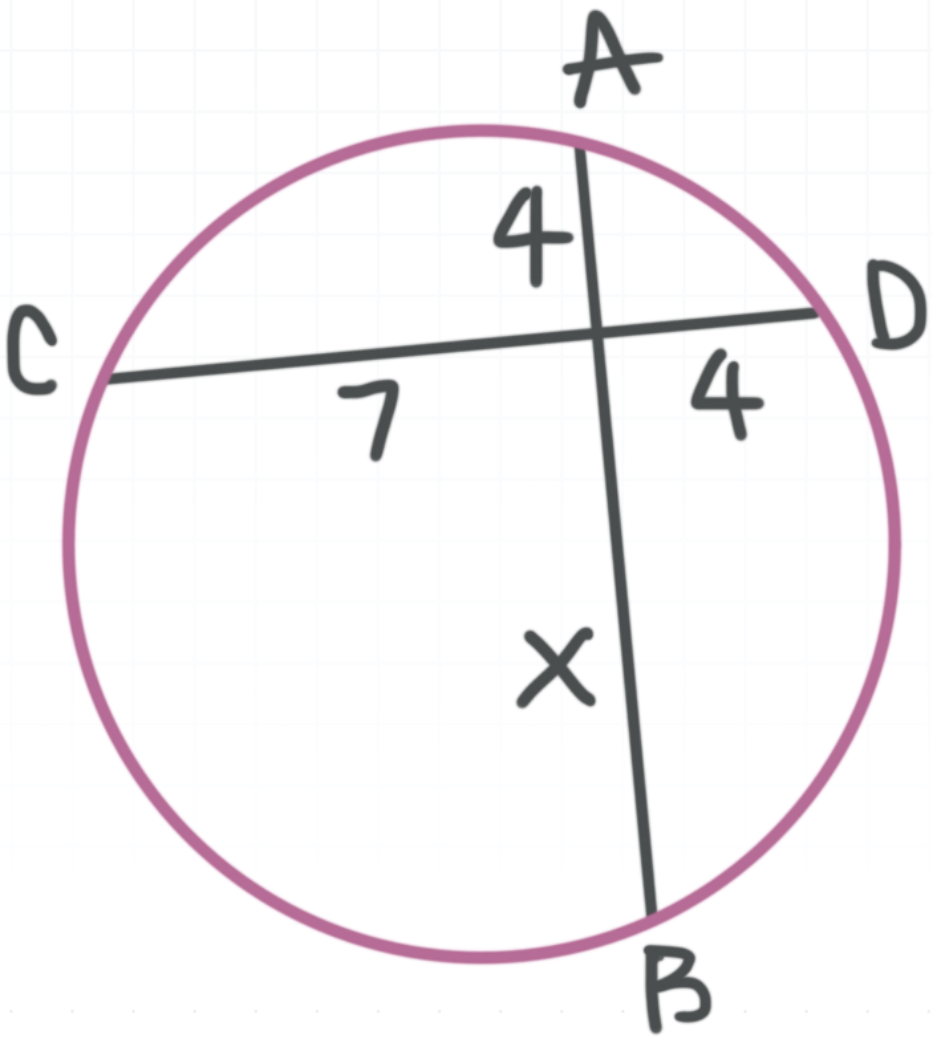
$$8(8 + 10) = 10(10 + x)$$

$$x = 4.4$$

© Krista King Math

INTERSECTING CHORDS

■ 1. $\overline{AB}$ and $\overline{CD}$ are intersecting chords of the circle. Find x.



Solution:

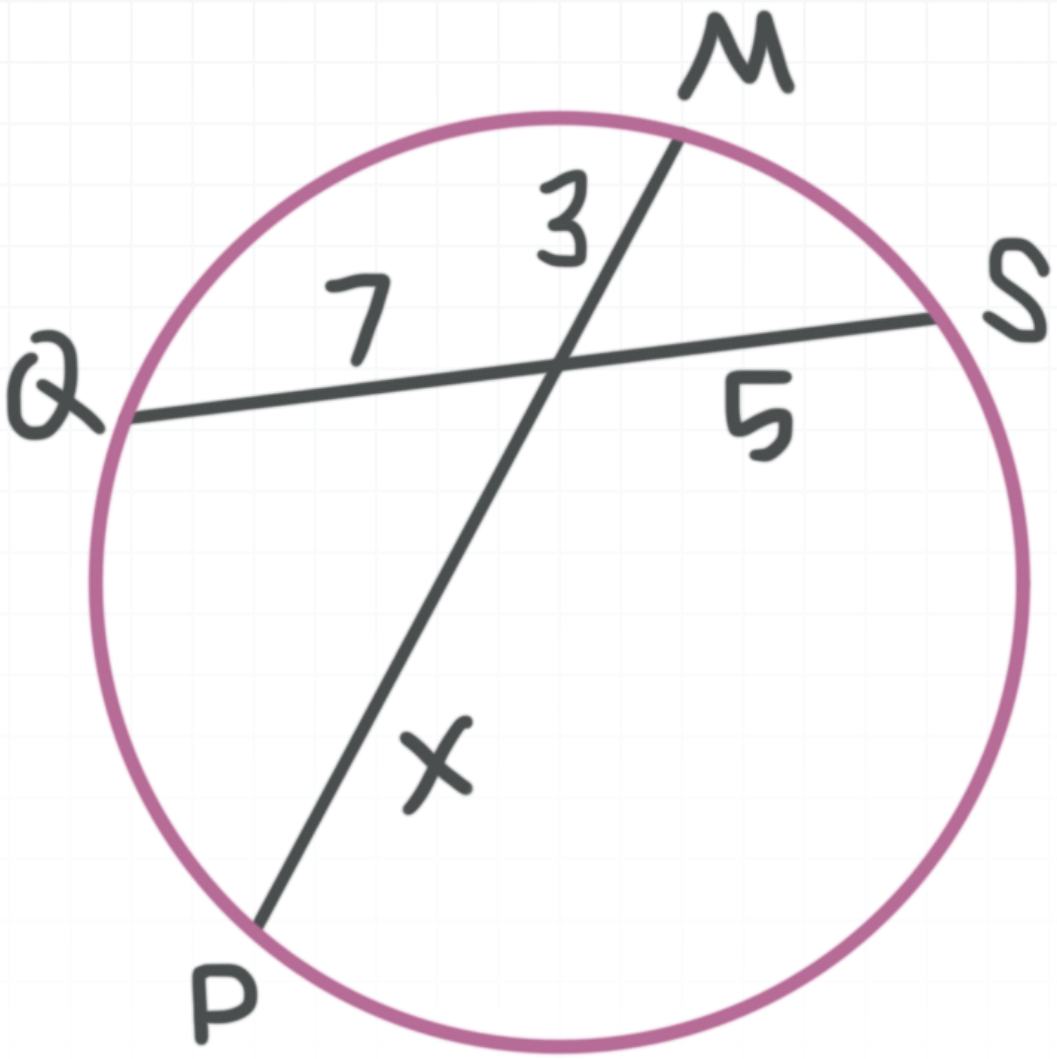
$x = 7$. From the figure, we know that

$$4x = 7(4)$$

$$x = 7$$

© Krista King Math

■ 2. $\overline{MP}$ and $\overline{QS}$ are intersecting chords of the circle. Find x.



Solution:

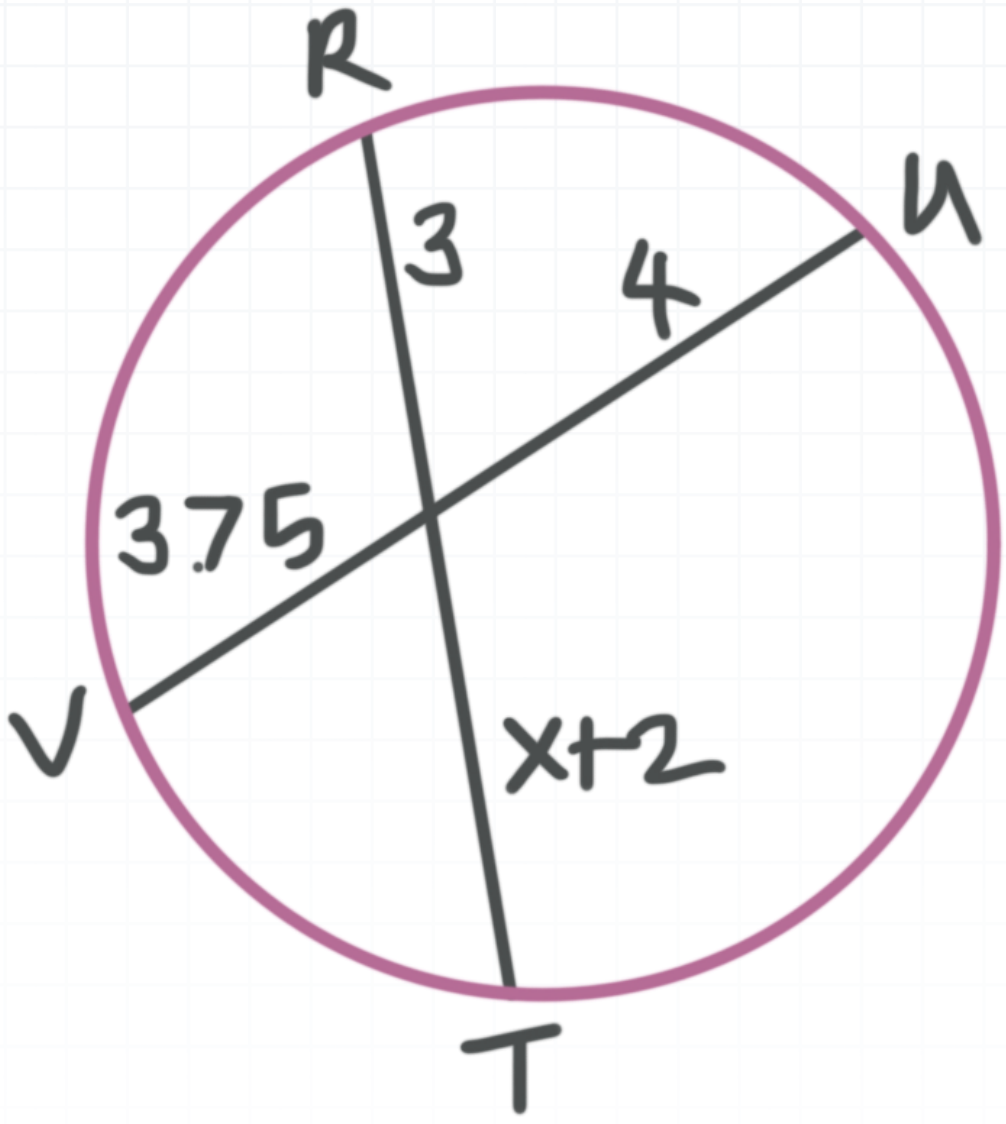
$x = 35/3$. From the figure, we know that

$$3x = (5)(7)$$

$$x = \frac{35}{3}$$

■ 3. $\overline{RT}$ and $\overline{UV}$ are intersecting chords of the circle. Find x.

© Krista King Math



Solution:

$x = 3$. From the figure, we can say

$$x(x + 2) = (3.75)(4)$$

$$x^2 + 2x = 15$$

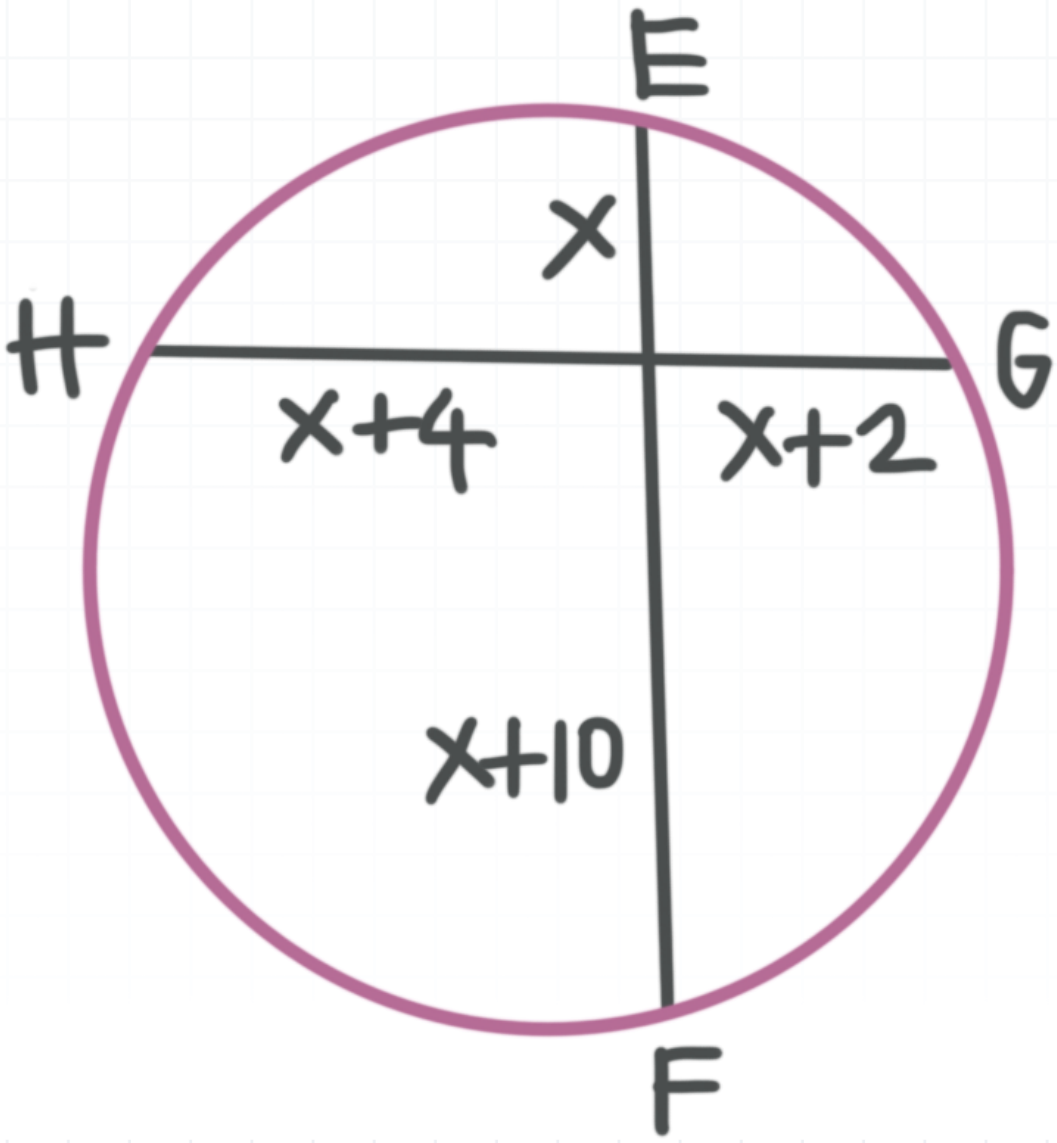
$$x^2 + 2x - 15 = 0$$

$$(x + 5)(x - 3) = 0$$

$$x = 3$$

© Krista King Math

4. $\overline{EF}$ and $\overline{HG}$ are intersecting chords of the circle. Find x.



Solution:

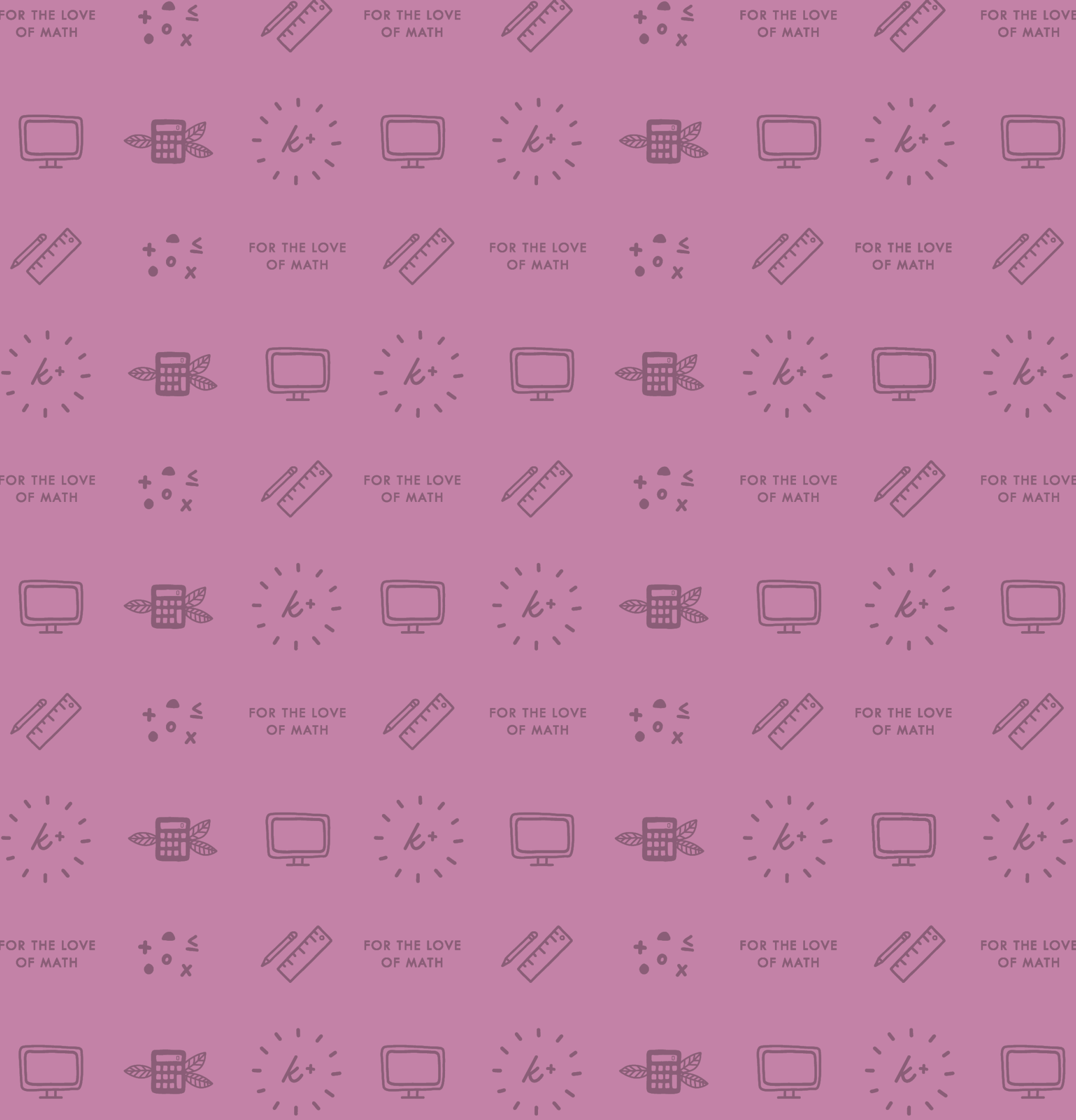
$x = 2$. From the figure, we know that

$$x(x + 10) = (x + 2)(x + 4)$$

$$x^2 + 10x = x^2 + 6x + 8$$

$$x = 2$$

© Krista King Math



WWW.KRISTAKINGMATH.COM